Value Tree Analysis

30.04.2002

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1. Introduction

1.1 Background

Complex, confusing decision problems with multiple objectives have been made since the start of the civilisation. The history of the decision analysis is not that long, however. In 1730s Daniel Bernoulli (1738) first used the concept of utility when explaining the evaluation of a particular uncertain gable known as St Petersburg paradox. He argued that money was not an adequate measure of value, but the worth of money for an individual was a non-linear function. The discovery created a base for the concept known as utility theory, a numerical measure describing the value of alternative choices, and utility function, the numerical measure itself. In the following century the concept of utility was mainly used to explain economic behaviour. Some utilitarian philosophers, such as Bentham and Mill also used the concept as a tool for constructing a theory of ethics. However, at that time it was not possible to measure person's utility function and the theory had only a limited importance in practice.

In 1940s and 1950s the utility theory was put on a sound theoretical foundation. Theory of games was developed to describe the behaviour of the rational people when engaging with others with conflicting goals. In 1944 John von Neumann and Oskar Morgenstern's **Theory of Games and Economic Behavior** was published, a book that made the most influential contribution to the development of modern decision theory. Sparked by the new theory, decision analysis emerged as researchers and practitioners mainly from the field of statistics and operations research developed prescriptive approaches and tools intended to help decision-makers (DMs) in difficult decisions.

Decision analysts (DAs) distinguished two types of utility. Value preferences are made between choices when no uncertainty is present. Risk preference addresses the DM's attitude towards risk taking under uncertainty. This learning package is concerned with choices under certainty, that is value theory, and specifically a decision analysis tool called value tree.

1.2 Uses of value tree analysis

Value tree analysis is an integral part of decision analysis (DA). In the following, main application areas with examples are listed.

Business, production and services:

- allocating budget

  How to allocate the annual engineering budget among products and projects? With value tree analysis aspects such as strategic fit, which have no natural evaluation measure, but may have a significant role in decision-making can be included into the analysis. Also, explicit modelling of the relevant facts is likely to increase communication and provide a base for justified decisions.

- selecting R&D programs

  In many R&D programs where risk is high, good reasoning may become as important as the decision itself. Value tree provides a tool for supporting the reasoning of the selection of the R&D programme and modelling the facts affecting the decision.

- developing and deciding on marketing strategies

  For example, analysing new strategies for merchandising gasoline and other products through full-facility service stations.

Public policy problems:

- analysing responses to environmental risks

  For example, structuring negotiations between several parties to identify compromise regulations for acid rain and identifying the objectives of the regulations.

- negotiating for oil and gas leases

  How to evaluate subcontractors? What criteria should be used?

- comparing alternative energy sources

- political decisions

  For example, structuring nuclear power debate, aiding the decision process, and studying value differences among the decision-makers.

Medicine:

- deciding on the optimal usage and inventory of blood in a blood bank

- helping individuals to understand the risks of different treatments

In addition to the decision-making problems value tree analysis serves also other purposes. It may be used for (von Winterfeldt & Edwards 1986)

- identifying and reformulating options

- defining objectives

- providing a common language for communication

- quantifying subjective variables

  For example a scale measuring the worth of military targets.

- developing value-relevant indices

  For example, an index describing the quality of water.

Readings

- Ulvila and Brown (1982)
- Corner and Kirkwood (1991)

1.3 Parties and roles in decision analysis

In decision analysis three different parties can be identified.

1. A **decision-maker** (DM) is a person, organisation or any other decision-making entity, who is empowered to make decisions concerning the decision-making problem at hand. In most cases the DM is also responsible for the decision and possible consequences.

2. A **decision analyst** provides insight and advice to the DM in difficult decisions. His / her task is to help the DM to find the most appropriate decision alternative(s) with possible reasoning and facilitate the decision-making process.

3. A **stakeholder** is a person or a body with an interest in the decision under consideration.

Possible relations between the different parties are described in Figure 1.3.1

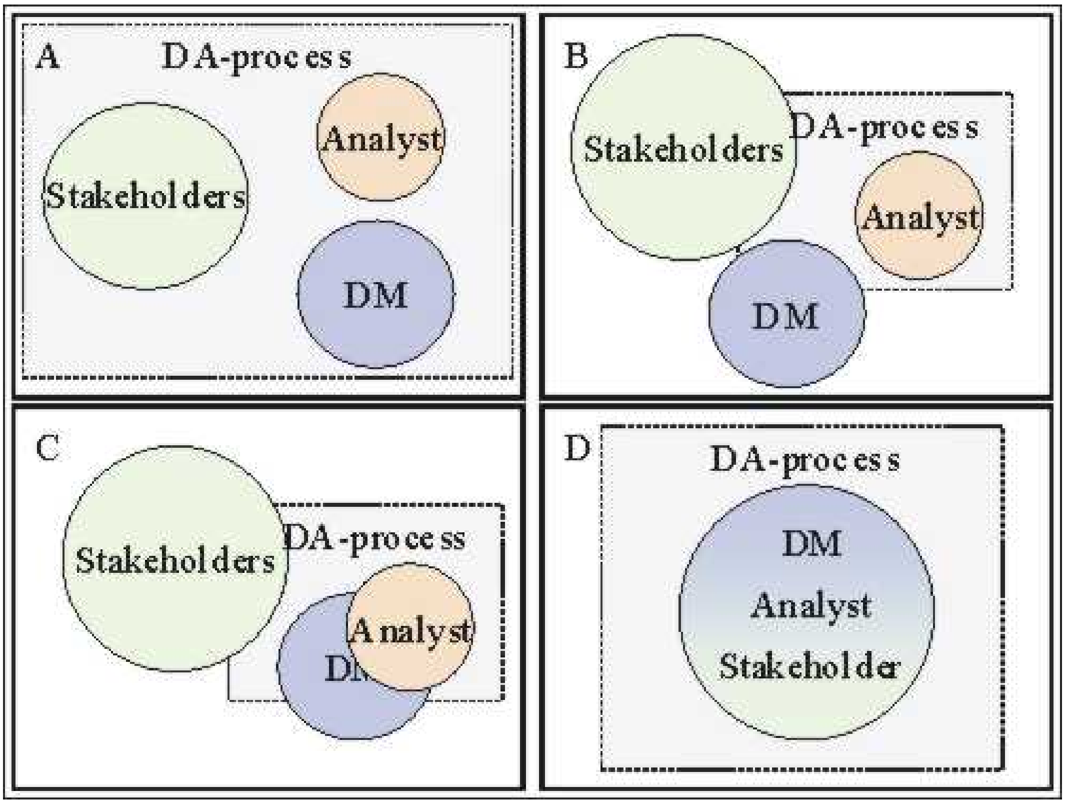


Figure 1.3.1 : Roles and parties in DA process.

It is worth noting that:

- Some key players are not necessarily included in the analysis. For example, it might take a considerable effort to identify all the stakeholder groups that may have only a little relevance to the decision analysis process (Figure 1.3.1 B).

- The roles of the DM, analyst and stakeholder may overlap. That is, they can partly represent the same body (Figure 1.3.1 C), or may even be a single person (Figure 1.3.1 D).

- As Figure 1.3.1 shows, the analyst can be a separate person, or body, or the DM can act as an analyst herself / himself.

1.4 The DA process

- The aim of the decision analysis (DA) process is to provide a structured way to think about decisions and develop and support subjective judgements that are critical for good decisions.

- As shown in Figure 1.4.1, DA processes typically involve four main phases.

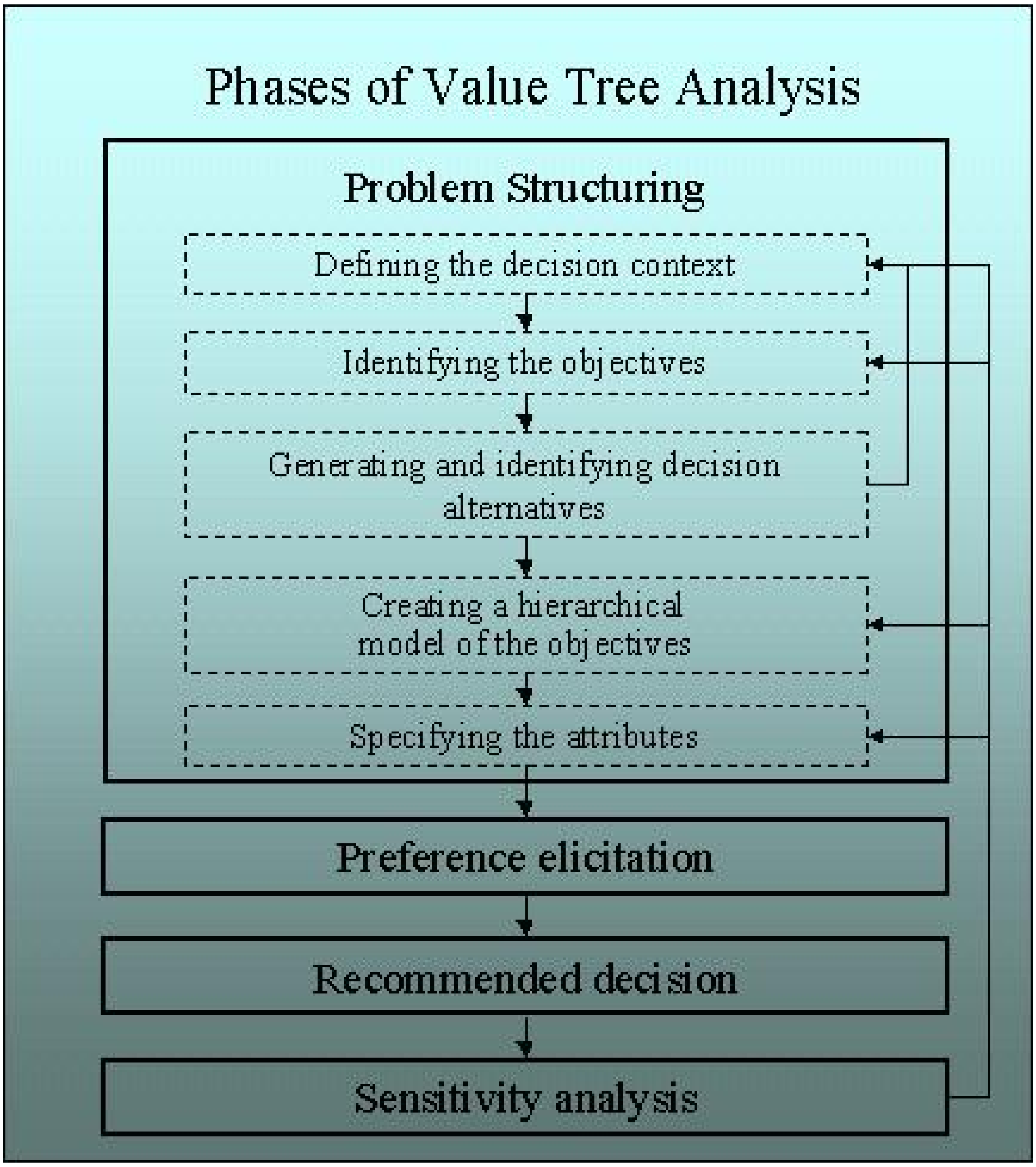


Figure 1.4.1 : Phases of the decision analysis process

- The processes are often large and iterative. For example, problem structuring, gathering of relevant information and the modelling of the DM's preferences often requires a considerable amount of work.

- The DM's perceptions of the problem as well as preferences for outcomes not considered before may change and develop during the process.

1.5 Problem structuring

- The main purpose of the problem structuring is to create a better understanding of the decision problem. For example, answers to the following questions should become evident.

- What is important, and relevant?
- What are the objectives?
- What is the real problem?
- Who are the parties involved?
- What information is available? Etc.

- After the decision situation and real nature of the problem is established, objectives and possible decision alternatives are identified.

- Relations between multiple objectives are analysed with hierarchical modelling. With a hierarchical model, relations between objectives are more easily understood. The model also creates a basis for further analysis.

- Attributes measure the extent to which different decision alternatives satisfy the stated objectives. Specification of attributes thus enables the comparison of the alternatives. For example, if sufficient compensation for the work done was an objective, salary measured in euros, could be a suitable attribute.

For detailed description of the phases of the problem structuring see the Problem structuring section.

1.6 Preference elicitation

- The aim of the preference elicitation is to measure and estimate the DM's preferences over a set of objectives.

- Measuring preferences is not straightforward. It may be that the DM is not sure about her preferences, she is unable to state them exactly, or she is even unaware of them. Furthermore, the DM may act inconsistently and give conflicting statements about her preferences.

- In most cases the preference elicitation is an iterative process in which several different methods may be used to ensure the best possible estimates of the DM's preferences.

- Knowing the DM's preferences, information about the attribute levels for different decision alternatives and the hierarchical model of the objectives can be used to find the most preferred alternative.

Different preference elicitation methods are described in the Preference elicitation section.

1.7 Sensitivity analysis

- The aim of the sensitivity analysis is to explore how changes in the model influence the decision recommendation.

- If a small change in one or several aspects of the model causes the recommended decision to change, the decision is said to be sensitive to those changes.
- Recognising the aspects to which the decision is sensitive enables the DM to concentrate on, or possibly reconsider the issues, which may cause changes in the decision.
- Any part of the decision analysis process, from the identification of the decision problem to the evaluation of the preferences, can be subjected to the sensitivity analysis.
- As Figure 1.4.1 shows, after the sensitivity analysis the DM may return to earlier phases of the DA process; new alternatives may be identified, model structure may be changed etc. Thus, sensitivity analysis is a central part of the decision analysis cycle.

For more information about sensitivity analysis see the Sensitivity analysis section.

1.8 A job selection problem

Assume that you have four job offers to choose between. The first offer is a place as a researcher in a Governmental Research Institute close to the city-centre, 45 minutes from your home. The head of the research department has sent you an offer letter in which he promises a starting salary of 1900€ a month with standard 37.5 weekly working hours and a permanent place in their research team. In the letter he also mentioned several training programs and courses related to the different research areas which are offered to the personnel. The job would be technically challenging, focused and gives opportunities for further studying. As there is no continuing need for domestic travelling the Research Institute does not provide their employees with company-owned cars. However, there are likely to be conferences all over Europe where you are assumed to attend every now and then (20 travelling days a year).

The second offer is from a multinational consulting firm. They have offered you a place for six months trial period, after which you could act as a junior consultant. The salary from the trial period is 2700€ per month, after which it is likely to rise to 3500€ in three years. According to the senior partner of the department, there is no reason to believe that they would not continue the work agreement after the trial period, but it is merely a matter of company's overall employment policy and your own will. The luxurious office of the company is located in the city-centre, 50 minutes from your home, but they have customers and departments all over Europe, where you are most likely to visit continuously (160 travelling days a year). All company's employees are young and they are expected to work hard 55 hours per week. The job would be neither highly technical nor too challenging, but it would include variable tasks and a substantial amount of management training. In the interview for the job, the senior partner also mentioned about social activities, such as golf club and courses, and company wide theme programmes which are set up to contribute employees' overall well-being. However, one of the consultants has told you know that only few of them were actually involved in those activities.

The third job offer is a place as a decision analyst in a large domestic firm. The office is located in an industrial area, less than one-hour travel from your home. The salary is 2200€ per month and the working time 8 hours a day. Also, a possibility to have a company-owned car is offered. The firm has a large number of active clubs and possibilities to do sports, and even a sports centre, which offers free services for all employees. Except the familiarisation period at the beginning, the job would not require or include further training or studying. However it would be challenging and include some variability and two to three day trips to the other domestic departments (100 travelling days a year). As opposed to the other job offers you would also have an own room with a view to the sea.

The fourth offer is from a small, promising, and fast growing IT firm established two years ago. The atmosphere is relaxed and employees are young, all under 35. The job description includes various activities from several areas of the business, some training, but only a limited amount of travelling (30 travelling days a year). The activities do not offer a great challenge, but most of them seem to be interesting. The salary is 2300€ per month and they expect you to work 42,5 hours per week and overtime if needed. The office is in the city centre, close to the bus station, which is about 40 minutes travel from your home. In the interview for the job they promised you a company-owned car and a possibility to use company's cottage close to a popular downhill skiing centre in the Alps.

As the firms differ considerably in their culture and atmosphere you decided to interview a couple of arbitrarily chosen employees from each firm. To ease the comparison of the opinions you asked the subjects to rate the atmosphere and corporate culture from 0 (poor) to 5 (very good). The results are shown in Table 1.8.1

Company	Average rating
Research Institute	3.2
Consulting Firm	2.5
Large Corporation	3.7
Small IT Firm	4.5

Table 1.8.1 : Corporate cultures and atmospheres

You have also come up with the following estimates for the expected salary in three years time.

Company	Expected salary in three years / €
Research Institute	2500
Consulting Firm	3500
Large Corporation	2800
Small IT Firm	3000

Table 1.8.2 : Expected salary in tree years

How would you approach the problem?

What would be the factors affecting your decision?

Do you think that there is any way to structure the problem?

2. Theoretical foundations

2.1 Concepts and notation

2.1.1 Objective

Definition 2.1.1.1 *(Keeney1992)*

Objectives are statements of something that one desires to achieve.

- Generally, objectives are characterised by three features (Keeney 1992):
  1. decision context
  2. object
  3. direction of preferences

  For example, with respect to job selection, one objective may be to maximise the compensation for work done. For this objective, the decision context is job selection, the object is compensation for work done, and more compensation is preferred to less compensation.

- Objective specification does not require the identification of a measure (for example salary in euros/month) to indicate the level to which the objective is achieved.

- An objective does not quantify the relative desirability of different levels of the object.

- Objectives can be divided into two classes (Keeney 1992):

  1. Fundamental objectives

     characterise an essential reason for interest in the decision situation.

  2. Means objectives

     are of interest in a decision context because they are means to achieving fundamental objectives.

  For example, higher salary may appear to be an important objective, but it may be seen important only because it would allow an individual to increase his/her living standard, to pursue activities that represent fundamental interests. Thus, higher salary could be seen as a means objective and *increase living standard* as a fundamental objective.

2.1.2 Attribute

- An objective is measured in terms of an **attribute**. That is, attribute $X_i(a) = x_i$ indicates the level to which the objective O_i is achieved in the alternative a. For example, *salary in euros per month* measures how the objective *compensation for work done* is achieved.

- The possible outcomes of the attribute are referred to as **performance levels**. When a performance level is associated to a certain alternative the term **consequence** is used instead.

- Attributes X_1, X_2,..., X_n create a mapping from the act space A into the n dimensional **consequence space**

  $$C = C_1 \times C_2 \times \ldots \times C_n$$

  where C_i, i=1,..., n is the set of possible levels of achievements measured with the attribute X_i.

- For a decision alternative a in A, the corresponding point in the consequence space is expressed as

  $$X(a) = (X_1(a), \ldots, X_n(a)) = (x_1, \ldots, x_n)$$

- The situation for **n = 3** is illustrated in Figure 2.1.2.1

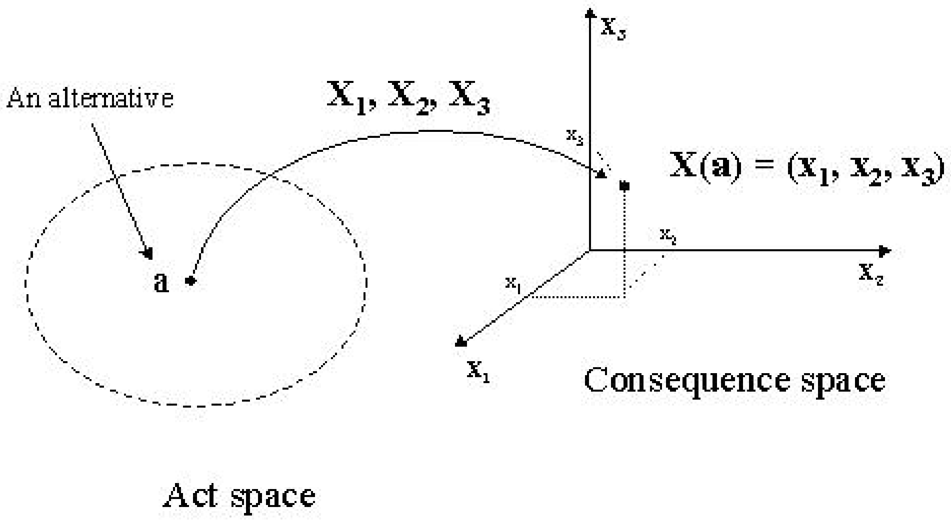


Figure 2.1.2.1 : The mapping of alternatives to the consequence space.

- For example, in the job selection case[1], the act space is a set of possible decision alternatives, that is job offers; attributes could be for example salary in euros per month, working hours per week, etc.; and the consequence space consists of outcomes of different job offers measured with the attributes.

2.1.3 Goal

- A goal is a specific level of an objective to be achieved. For example, for "compensation for work done" objective, a goal could be "more than 1700€ per month".

- As opposed to an objective a goal is either achieved or not.

2.1.4 Preferences

- Notation

  $a \succ b$

  means that the DM strictly prefers the object a to the object b. In other words, if a choice between a and b was offered to her, she would be disappointed if she then had to select the object b.

- Notation

  $a \succsim b$

  means that the DM **weakly prefers** the object a to the object b. That is, according to the DM the object a is at least as good as the object b. If

[1]http://www.mcda.hut.fi/value_tree/cases/Job/slides/

the DM was offered a choice between the object a and b, she would not be disappointed if she then was forced to take the object a.

- Notation

  $a \sim b$

  means that the DM is **indifferent** between the object a and b. In other words, if she was offered a choice between a and b, she would not be disappointed if she was subsequently forced to take either of the options.

- If a ~ b, a, $\in$ C^2 they are said to be on the same **indifference curve.** If a, b $\in$ C^n, n>2 they are said to lie on the same indifference surface. Thus, indifference curve, or surface is a geometric presentation of an indifference set I = $\{x_i, i \in J \subset N \mid x_i \sim x_j, i, j \in J\}$.

- Weak preference, strict preference and indifference are examples of concepts known as binary relations. For more information about binary relations and preferences see French (1988) or Fishburn (1970).

2.1.5 Value function

- A value function v assigns a number v(x) to each consequence x = $(x_1,..., x_n)$, where x_i is a level of attribute X_i measuring object O_i such that the numbers v(x)

  1. indicate the relative desirability of the consequence , and

  2. can be used to derive preferences for alternatives.

- Value functions enable a compact representation of preferences. For describing preferences over n objects only n real numbers are required; the object with a greater value is preferred to objects with smaller values.

- A scalar-valued function v defined on the consequence space with the property

  $$v(\mathbf{X}_1) \geq v(\mathbf{X}_2) \Leftrightarrow \mathbf{X}_1 \succsim \mathbf{X}_2 \qquad \mathbf{X}_1, \mathbf{X}_2 \in C$$

  is called an **ordinal value function.**

- A scalar-valued function v defined on the consequence space C is called a **measurable value function** (value difference function, cardinal value function), if

  $$v(\mathbf{X}_1) \geq v(\mathbf{X}_2) \Leftrightarrow \mathbf{X}_1 \succsim \mathbf{X}_2$$

  and

$$v(\mathbf{X}_1) - v(\mathbf{X}_2) \geq v(\mathbf{X}_3) - v(\mathbf{X}_4) \Leftrightarrow (\mathbf{X}_1 \leftarrow \mathbf{X}_2) \succsim_e (\mathbf{X}_3 \leftarrow \mathbf{X}_4)$$

$$\mathbf{X}_i \in C, \mathrm{i} = \{1,..,4\}$$

where the last equation indicates that the DM considers the exchange of X_2 to X_1 at least as good as the exchange of X_4 to X_3.

- Ordinal value functions capture the preference order information but do not say anything about the strength of the preferences. Consider the following situation.

$$\begin{cases} v(\mathbf{X}_1) \geq v(\mathbf{X}_3) \\ v(\mathbf{X}_2) \geq v(\mathbf{X}_3) \end{cases} \Leftrightarrow \begin{cases} \mathbf{X}_1 \succsim \mathbf{X}_3 \\ \mathbf{X}_2 \succsim \mathbf{X}_3 \end{cases} \Rightarrow \quad \mathbf{X}_1 \ ? \ \mathbf{X}_2$$

- Measurable value function describes also the strength of the preferences.

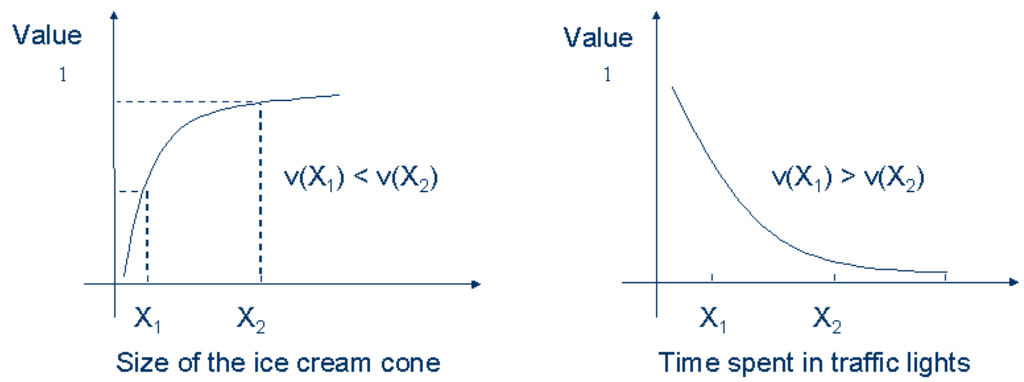


Figure 2.1.5.1 : Measurable value functions

2.2 Axiomatic foundations

In decision theory, the DM is assumed to behave rationally. These assumptions are expressed in axioms, on which the whole theory is built. The axioms can be stated as follows (French 1988).

Let A be a set of objects over which the DM's preferences are expressed.

1. Comparability

$$\forall\, a, b \in A,$$
$$\begin{cases} a \succsim b, & \text{or} \\ b \succsim a, & \text{or} \\ a \sim b & \end{cases}$$

In other words, comparability states that the DM is able to compare any objects in terms of his preferences, i.e. she/he is not indecisive.

2. Transitivity

$$\forall\, a, b, c \in A,\ \text{if } a \succsim b \text{ and } b \succsim c \Rightarrow a \succsim c$$

3. Consistency of indifference and weak preference

$$\forall\, a, b \in A \quad a \sim b \Leftrightarrow (a \succsim \mathrm{b} \text{ and } b \succsim \mathrm{a})$$

Thus, a rational DM is indifferent between the objects a and b if and only if each of them is at least as good as the other.

4. Consistency of strict preference and weak preference

$$\forall\, a, b \in A \quad a \succ b \Leftrightarrow \neg(b \succsim \mathrm{a})$$

That is, the DM strictly preferes a to b if and only if she/he does not think that b is at least as good as a.

If, for some reason, the axioms do not seem meaningful in a particular context, methods based on the value theory should not be used in decision analysis, but other approaches should be used instead. From the axioms several results can be derived. For example,

$$a \sim b \text{ and } b \succ c \Rightarrow a \succ c$$

For a finite set of objects A = $\{a_1, a_2, \ldots, a_n\}$ with a weak preference order obeying Axioms 1-4, there is an agreeing ordinal value function v(a), such that

$$v(a_i) \geq v(a_j) \Rightarrow a_i \succsim a_j$$

Thus, from the axioms immediately follows the existence of the **ordinal value** function.

With some additional assumptions also the existence of a measurable value function, which describes the strength of the preferences can be guaranteed.

Readings

- French (1988)
- Fishburn (1970)

2.3 Strategic equivalence

Definition 2.3.1 ***(Keeney1976B)***

The value functions v_1 and v_2 are strategically equivalent, written $v_1 \sim v_2$, if v_1 and v_2 have the same indifference curves and induced preferential ordering

- For example, value functions

  $$v(\mathbf{x}) = \sum_i k_i x_i, \quad v(\mathbf{x}) = \sqrt{\sum_i k_i x_i}\ , \quad \text{and} \quad v(\mathbf{x}) = \log\left(\sum_i k_i x_i,\right)$$

  are strategically equivalent, since they imply the same preference structure.

- The same argument can be used to show that positive affine transformations of a value function are strategically equivalent. That, is

  $$v(\mathbf{x}) \sim \alpha + \beta v(\mathbf{x})$$

  where α and β are constants and $\beta > 0$.

- Consequently, there is no absolute value scale. For example, value "50" has no interpretation without the context and all value functions can be scaled to give outcomes within the desired range. Most commonly values are scaled either between 0 and 1, or between 0 and 100.

- To conclude, a value function uniquely specifies the entirely preference structure, but a preference structure does not uniquely specify a value function.

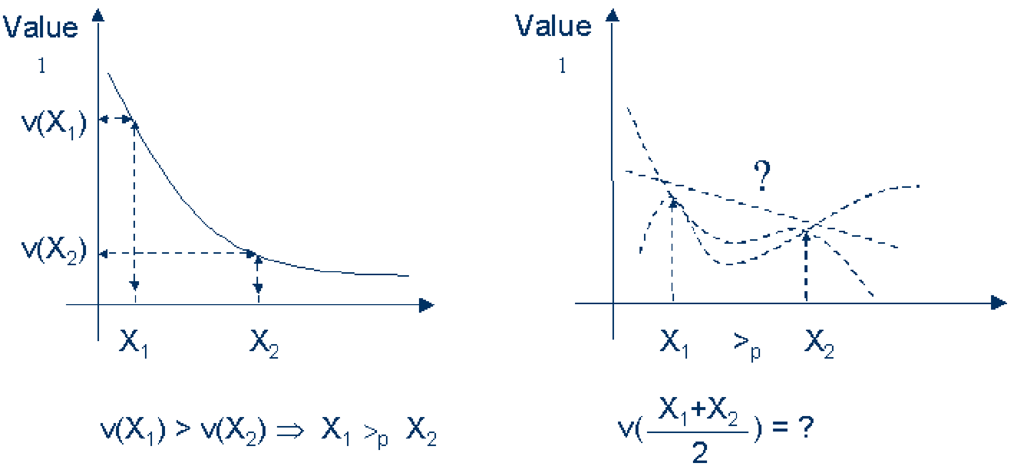


Figure 2.3.1 : Value function and preferences

2.4 Mathematical representation of the decision problem

Let a denote a feasible decision alternative, and A a set of all feasible alternatives. Furthermore, assume that for each alternative a in A, n attributes X_i, i = {1,..,n} and a consequence X(a) = (X_1(a),...,X_n(a)) are associated.

To find the best alternative for a decision problem, the DM has to choose a in A such that she will be happiest with the consequence X(a). Thus, the decision problem can be formulated as:

Find $a^0 \in$ A such that

$$\mathbf{X}(a^0) \succsim \mathbf{X}(a), \quad \forall\, a \in A$$

where

$$\mathbf{X}(a) = \left[\mathbf{X}_1(a), \ldots, \mathbf{X}_n(a)\right]$$

If an n dimensional value function v reflects DM's preferences, the decision problem can be stated as a standard optimisation problem:

$$\max_{a \in A} v(\mathbf{X}(a))$$

The mathematical presentation of the decision problem is in keeping with the formulation presented in Keeney and Raiffa (1976).

2.5 Decomposition

The aim of decomposition is to express the total value of a decision alternative a and the corresponding consequence x=(x_1,..., x_n) with values of attribute levels X_i(a)=x_i. Formally,

$$V(x_1,\ldots,x_n) = f\left(v_1(x_1),\ldots,v_n(x_n)\right)$$

In the following value models, it is assumed that

$$v_i : C_i \to [0, 1] \quad i = \{1,..,n\},\ C = C_1 \times C_2 \times \ldots \times C_n$$

where C is the consequence space. That is, all single attribute value functions v_i have the range [0, 1].

The assumption can be made, because positive affine tranformations of value functions are strategically equivalent and induce the same preference order.

2.5.1 Additive model

- In the additive model the total value V is of the form

  $$V(x_1,\ldots,x_n) = \sum_{i=1}^{n} w_i v_i(x_i)$$

  where v_i is a single attribute value function over the consequence x_i and w_i is the corresponding weight.

- The weight w_i corresponds to the change in the strength of preferences as the attribute X_i changes from the worst to the best level.

- The weights are often normalised in such a way that the sum of the weights equals one.

- The additive model describes the DM's preferences only if the attributes are mutually preferentially independent. (See the Preference independence section.) Thus, there cannot be synergies between the attributes.
  - For example, a modern management information system is highly valuable if there are knowledgeable persons to utilise it. In that case, there are likely to be synergies between the information systems and the education level of the personnel, which cannot be captured with an additive model.

- There cannot be threshold levels for any attribute.
  - For example, in many industrial processes a certain amount of energy is required to run the production. If there is short of energy the value of process related resources are close to zero while value cannot be created.

- While the structure is simple and easily understood, an additive value model is assumed in most of the cases in practice.

- If the preference independence cannot be assumed, some other model should be used instead.

2.5.2 Multiplicative model

- In the multiplicative model the total value is assumed to be of the form

$$V(x_1,\dots,x_n) = \frac{1}{K}\left[\prod_{i=1}^{n}\left(Kk_i v_i(x_i)+1\right)-1\right]$$

where K is a nonzero solution to the equation

$$K+1 = \prod_{i=1}^{n}\left(1+Kk_i\right)$$

and k_i is the value of an outcome having the best level on the attribute X_i and worst on all others.

- In the model single attribute value functions have multiplicative effect on the total value.

2.5.3 Multilinear model

- In the multilinear model the total value is assumed to be of the form

$$V(x_1,\dots,x_n) = \sum_{i=1}^{n} w_i v_i(x_i) + \sum_{i=1}^{n}\sum_{j>i} k_{ij} v_i(x_i) v_j(x_j)$$

- With a multilinear model, multiplicative interactions between the attributes can be added to the additive model.

2.6 Preference independence

Definition 2.6.1

Attribute X_1 is **preferentially independent** of attribute X_2 if for all $x_1, x_1' \in X_1$

$$(x_1,\alpha) \succsim (x_1',\alpha)$$

for some $\alpha \in X_2$

$$\Rightarrow (x_1,\beta) \succsim (x_1',\beta)$$

for all $\beta \in X_2$

Thus, if the attribute X is preferentially independent of the attribute Y, preferences for specific outcomes of X do not depend on the level of the attribute Y.

Definition 2.6.2

Attributes $X_1,\ldots, X_n$ are **mutually preferentially independent** if all subsets $S' \subseteq S=\{X_1 \times X_2 \times \ldots \times X_n\}$ are preferentially independent of their complement S'^c in $\{X_1 \times X_2 \times \ldots \times X_n\}$.

Consider the following example.

A person is making a decision about moving to a new house and buying a new car. Let the attribute X denote the location of the new house, and the attribute Y denote the make of the car. The possible values of the attributes are

X: {x_1=Helsinki, x_2=a dessert in Africa}

Y: {y_1=Ferrari, y_2=Jeep}

Suppose

$$\begin{cases}(x_1, y_1) \succ (x_2, y_1) \\ (x_1, y_2) \succ (x_2, y_2)\end{cases}$$

That is, Helsinki is always preferred to Africa, irrespective of the car in question.

However, it may well be that

$$\begin{cases}(x_1, y_1) \succ (x_1, y_2) \\ (x_2, y_2) \succ (x_2, y_1)\end{cases}$$

indicating that the person prefers the Ferrari if the new house is in Helsinki, but if the new house is in Africa she considers the Jeep as a better option.

If (3) and (4) hold, X is preferentially independent of the attribute Y, but Y is not preferentially independent of the attribute X. Thus, the attributes are not mutually preferentially independent.

3. Problem structuring

3.1 Phases of problem structuring

There are two general approaches to problem structuring:

1. first identify the problem and then figure out the appropriate objectives
2. first understand the values and objectives and then look for decision opportunities

The former approach emphasises problem focused thinking, and the latter value focused thinking.

In the following illustration 3.1.1 a possible problem focused approach to the problem structuring is illustrated.

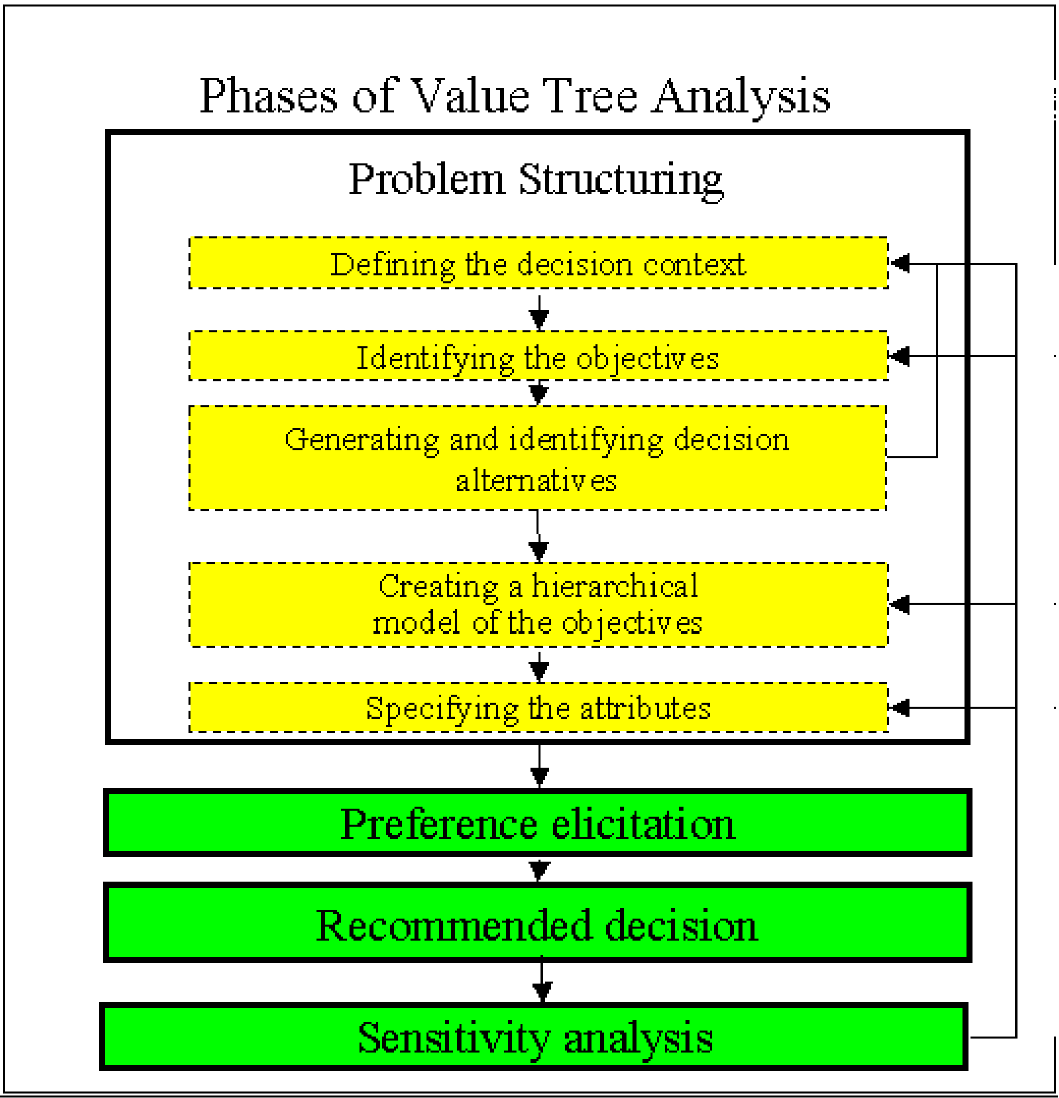


Illustration 3.1.1 : Phases of problem structuring.

Problem structuring is one of the most important parts of the value tree analysis. It gives a

- better understanding of the problem
- better understanding of the values affecting the decision
- basis for further analysis
- common language for communication

3.2 Defining the decision context

Definition 3.2.1

Decision context is the setting in which the decision occurs. It is framed by the administrative, political and social structures that surround the decision under consideration. Most readily it is specified by the activity being contemplated.

In Figure 3.2.1, main factors and questions specifying the decision context is shown.

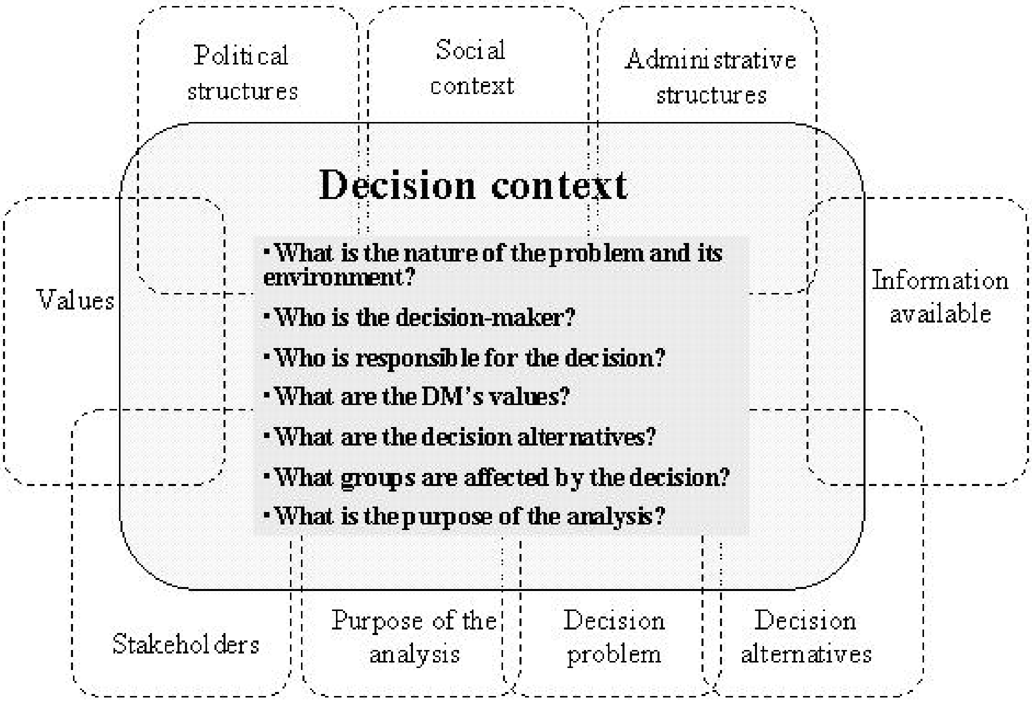


Figure 3.2.1 : Decision context

- Decision context and corresponding fundamental objectives are closely related and they frame the **decision situation** . For example, one decision context facing you may be to decide where to go for lunch. Clearly the objectives are different from the situation when considering different career opportunities.
- By defining the decision context and establishing the nature of the decision problem carefully, the treatment of the real problem can be ensured.
- A careful specification of the decision context is particularly relevant if several DMs or stakeholders are involved in the decision analysis process. Without a mutual agreement on the decision context problems

are likely to occur in the subsequent phases.

- Note that a decision context may be a decision alternative in a broader decision context. For example, consider a job selection problem within a given industry, in certain country or area, job selection based on certain competence base, or selection of a life style.

See the Job selection case - Defining the decision context[2].

3.3 Identifying and generating objectives

Identifying objectives requires significant creativity. Thus, an analyst often has an important role as a facilitator in guiding and stimulating the process.

The most obvious way to identify objectives is to ask a group of decision-makers or stakeholders first recapitulate the decision context, then individually provide a written list of objectives and then move on to a group discussion of the lists.

Several devices can be used to stimulate the identification (Keeney 1992).

1. A wish list

   The idea is to list all possible objectives without ranking or priorisation.

2. Use of alternatives

   The facilitator can ask DMs to identify the features that distinguish existing or hypothetical alternatives.

3. Use of problems and shortcomings

   Major problems are often related to objectives. By identifying the shortcomings and reasons for concern specific objectives to alleviate these problems can be found. Alternatively, DMs can be asked how matters could or should be improved, or why they are less satisfied with some reason than the other.

4. Use of consequences

   Consequences indicate the degree to which objectives are met. Thus, by asking DMs to articulate consequences, associated objectives may be found more easily.

5. Use of goals, constraints and guidelines

   Both goals and constraints are closely related to objectives. Goals state what to do whereas constraints state what not to do. By asking the objectives that led to the establishment of a goal or a constraint may help to identify the objectives for the problem under consideration.

[2]http://www.mcda.hut.fi/value_tree/cases/Job/slides/sld013.htm

6. Use of different perspectives

   Normally people think the objectives from their own perspective. By asking them to take the perspective of some other stakeholder new objectives may be found. Also, the current situation may be viewed from the future. Where you would like to be in ten years, and how it is related to the current situation? Furthermore, some of the realism can be eliminated from the current situation. For example, respondents can be asked to suppose that they can act without any limitation or consequences.

7. Structuring objectives

   By structuring the objectives and studying the relations and interactions between them is likely to stimulate the generation process. Also identifying the means and fundamental objectives with the specification of attributes gives more insight into the problem and may lead to the identification of new objectives. Hierarchical modelling of the objectives and attribute specification is discussed in detail in the following chapters.

Also, totally different approaches can be taken to the generation of the objectives (Keeney and Raiffa 1976) :

1. Examination of the literature

   By studying problems similar to the one under consideration relevant objects may be found.

2. Analytic study

   By building a model of the system under consideration and identifying relevant input and output variables, suitable objectives become obvious.

3. Casual empiricism

   Objectives may be generated by observing people who are making decisions that are relevant to the problem.

4. Surveys

   In public decision-making individuals affected by the decision may be asked what objectives should be included in the study.

5. Expert panel

   A group of people with expertise in the area may be used to generate the objectives.

See the Job selection case - Generating objectives[3].

[3]http://www.mcda.hut.fi/value_tree/cases/Job/slides/sld015.htm

3.4 Generating and identifying decision alternatives

As in the objective generation process, a possible way to identify and generate decision alternatives is to ask a group of decision-makers or stakeholders individually provide a written list of alternatives and then move on to group discussion of the lists.

Several devices can be used to stimulate the creation of alternatives (partly adapted from Keeney 1992).

1. Use of fundamental objectives
   - What would be the most desirable alternative if there were only one specific objective?
   - What would be the most desirable alternative if there were two given objectives?
   - Continue until all objectives are considered together.

2. Use of means objectives
   - When creating alternative means objectives can be used instead of fundamental objectives.

3. Removing constraints
   - Removing constraints on alternatives, or consequences may also create desirable alternatives.
   - What would be the most desirable alternative if cost were no concern?

4. Using different perspectives
   - What would be the most desirable alternative from a specific stakeholder's point of view?

3.5 Hierarchical modelling of objectives

The aim of the structuring and hierarchical modelling of the objectives is to create a deeper and more accurate and analytic understanding of the problem and a basis for quantitative analysis.

Hierarchical modelling of objectives is described in detail in the sections

- Separating means from fundamental objectives
- Objectives structures

- Constructing objectives structures
- Checking the structure

3.5.1 Separating means from fundamental objectives

As the major goal of the objective generation process is to produce an exhaustive list of objectives, they are likely to be inconsistent and vary in their scope, explicitness and detail. For that reason structuring and apportionment to fundamental and means objectives is required.

Fundamental and means objectives have different roles in the analysis:

- Fundamental objectives characterise the reason for interest in a decision situation, and thus are essential part of the problem structuring.
- Means objectives are helpful for creating alternatives and developing models to analyse the decision problem.

Means and fundamental objectives can be separated by asking: "Why is this objective important in the decision context?"

- Means objectives are important because of their implications for other objectives.
- Fundamental objectives are important because they are an essential reason for interest in that situation.

3.5.2 Objectives structures

In literature, objectives structures often include both fundamental and means objectives. Furthermore, in many cases the relations between the objectives are not clearly specified. Here we make the following distinction between the objective structures (Keeney 1992) .

Fundamental objectives hierarchy:

- The hierarchy includes only fundamental objectives.
- A higher-level objective is defined by the set of lower-level objectives under it.
- Within any set, the lower-level objectives are mutually exclusive and provide an exhaustive characterisation of the higher-level objective.
- Every higher-level objective has at least two lower-level objectives connected to it.

Note: Here we use the term **value tree** when referring to the fundamental objectives hierarchy and attributes associated with it.

In Figure 3.5.2.1 a fundamental objectives hierarchy related to the safety of automobile travel is shown.

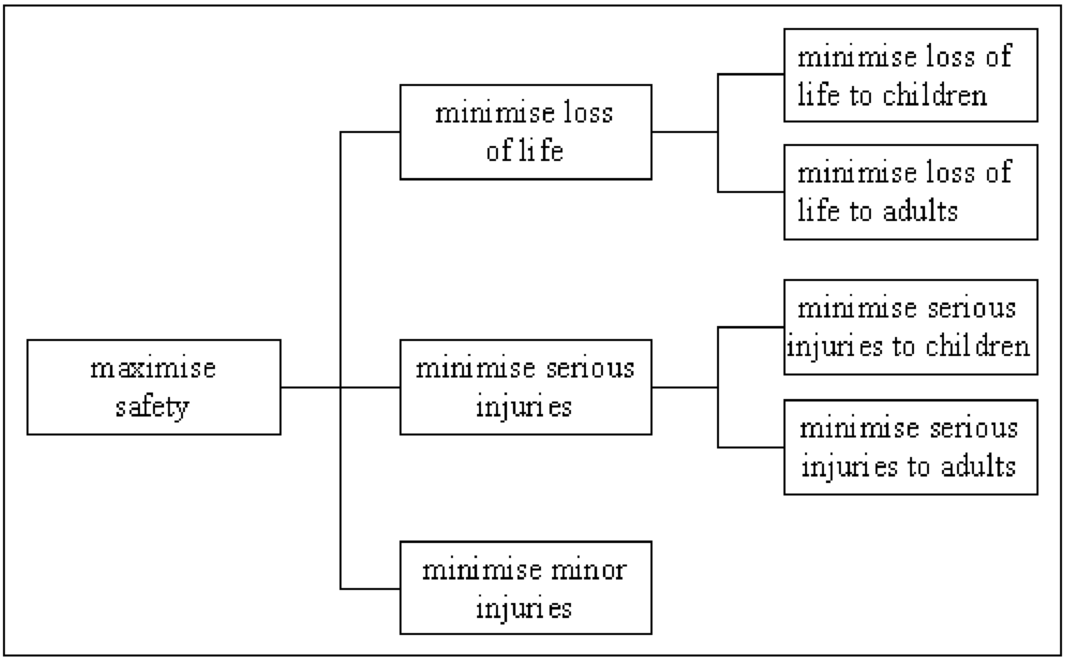


Figure 3.5.2.1 : Fundamental objectives hierarchy (Keeney 1992, p.70, Figure 3.1 a)

Means-ends objectives network:

- The network may include both fundamental and means objectives.
- A lower-level objective is a means to the higher-level objective.
- The set of means objectives under a higher-level objective does not necessarily provide an exhaustive representation of the means leading to the higher-level objective.
- A higher-level objective may have only one lower-level objective connected to it.

In Figure 3.5.2.2 means-ends objectives network related to the safety of automobile travel is shown.

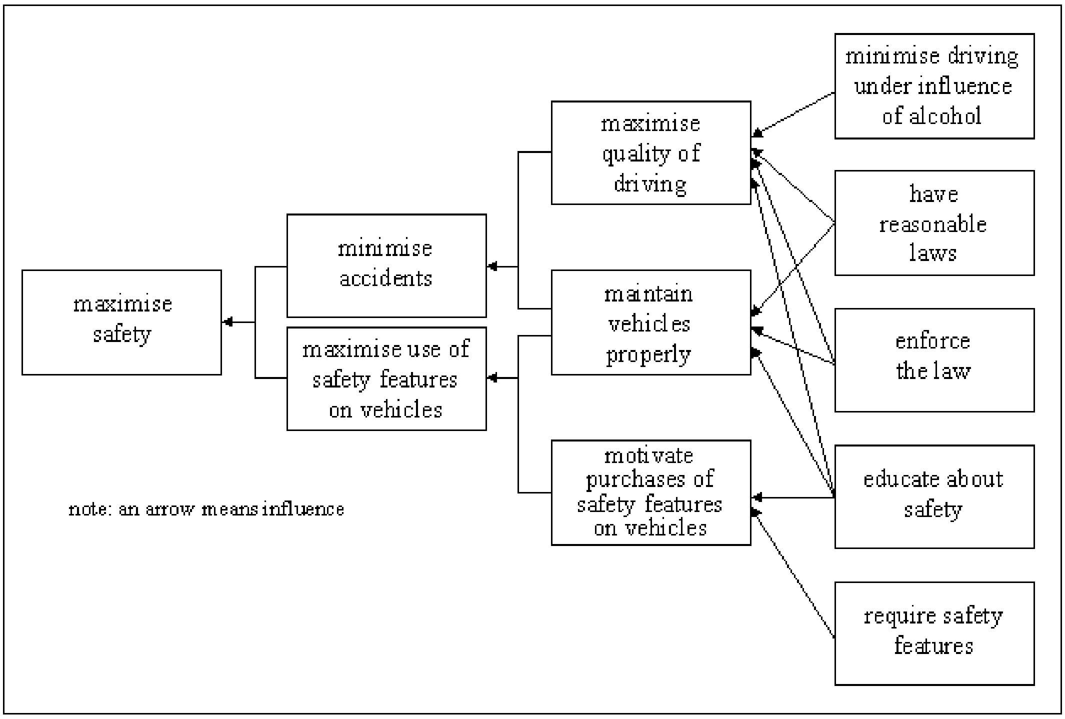


Figure 3.5.2.2 : Means-ends objectives network (Keeney 1992, p.70, Figure 3.1 b)

3.5.3 Constructing objectives structures

There are two ways to construct objectives structures

1. A **top-down approach** starts from the most general objective, which is then successively divided into sub-objectives.

2. In a **bottom-up approach** all meaningful differences between alternatives are first listed and then combined and structured to higher level objectives.

In general, the top-down approach is most appropriate when constructing a fundamental objectives hierarchy and the bottom-up approach is most suitable when generating a means-ends objectives network. In the following, the top-down approach is presented.

Top-down approach

1. Identify the overall fundamental objective.
   - In many cases the overall fundamental objective is obvious from the decision context. For example, the essence of a financial investment is to make money.

   - The overall objective may be a combination of more specific fundamental objectives. In that case, the analyst can ask the DMs to list relevant general values or important fundamental objectives. Dividing the list into categories should provide a basis for defining the overall fundamental objective and a basic structure for the objectives hierarchy.

2. Specify and clarify the intended meaning of the objectives in terms of more specific objectives.
   - The analyst can ask the DMs to state what aspects of the higher-level objectives they consider as important?

3. Subdivide the objectives until the lowest level is sufficiently well defined that a measurable attribute can be associated with it.

3.5.4 Checking the structure

When constructing the objectives hierarchy the analyst should check that (adapted from Von Winterfeldt and Edwards 1986 and Keeney 1992)

1. The division of an objective into lower-level objectives is reasonable, that is, the division clarifies the meaning of the upper-level objective and the relation between them is hierarchical.

2. There are no unnecessary cross-links between a set of lower-level objectives and upper-level objectives. That is, the set of lower-level objectives should be unique to the upper-level objective.

3. The set of objectives is exhaustive and nonredundant.

4. The set of objectives is **essential**. That is, each of the alternatives included in the decision context can influence the degree to which the objectives are achieved.

5. The set of objectives is **controllable**. That is, all the decision alternatives that can influence the degree to which the objectives are achieved are included in the decision context. This condition may be difficult to achieve, however.

See the Job selection problem - Hierarchical organisation of objectives [4] .

3.6 Specification of attributes

The degree to which objectives are achieved in different decision alternatives is measured with attributes. For example, the objective of a person to maximise her/his income can be measured with the attribute "salary in euros per month".

There are three types of attributes

1. Natural attributes
   - Natural attributes can be measured in natural scale, in centimetres,

[4]http://www.mcda.hut.fi/value_tree/cases/Job/slides/sld019.htm

dollars, numbers etc, and they have a common interpretation to everyone.

2. Constructed attributes
    - Constructed attributes do not necessarily have a common interpretation.
    - In most cases they are developed for a given decision context.
    - For example, the objective "maximise the positive impact on working environment" can not be measured explicitly with any single natural measure. However, it is possible to construct an attribute with levels say, from 0 to 5 describing the impacts. Clearly, the measurement is subjective.

3. Proxy attributes
    - Proxy attributes do not measure **directly** the degree to which fundamental objectives are achieved.
    - Level of proxy attributes should be valued only for their perceived relationship to the achievement of the corresponding fundamental objective.
    - For example, firms may have objectives such as prestige or power. For those objectives it is difficult to find natural or constructed attribute. However, "share of the market" may be used as a proxy attribute to measure **indirectly** the effects the growth potential of a firm.

In general, natural attributes should be preferred to constructed and proxy attributes.

Sometimes it is difficult to find appropriate natural, constructed or proxy attributes. In that case it is possible to use **direct preference measurement**. In direct preference measurements no attribute scale is constructed, but the effects of decisionalternativeson an objective or anattributeis assessed directly.

Attributes should be

1. Comprehensive and understandable
    - By knowing the level of an attribute the DM should have an unambiguous understanding of the extent to which the objective is achieved.
    - There should be no ambiguity in describing the level of which an objective is achieved in terms of an attribute.

2. Measurable

- It is possible to assess the DM's preferences for different levels of the attribute.
- Measuring the DM's preferences over the different levels of the attribute should be possible also in practice, that is without excessive amount of time, money and effort.

After the value tree is constructed each decision alternative is assessed in a performance matrix.

In Figure 3.6.1 the performance matrix of a value tree evaluating the performance of five old computers is shown. The performance matrix is constructed with the Web-Hipre software.

Note that Web-HIPRE uses the term rating when referring to a consequence . In literature also terms performance levels, achievement levels and measurements are used.

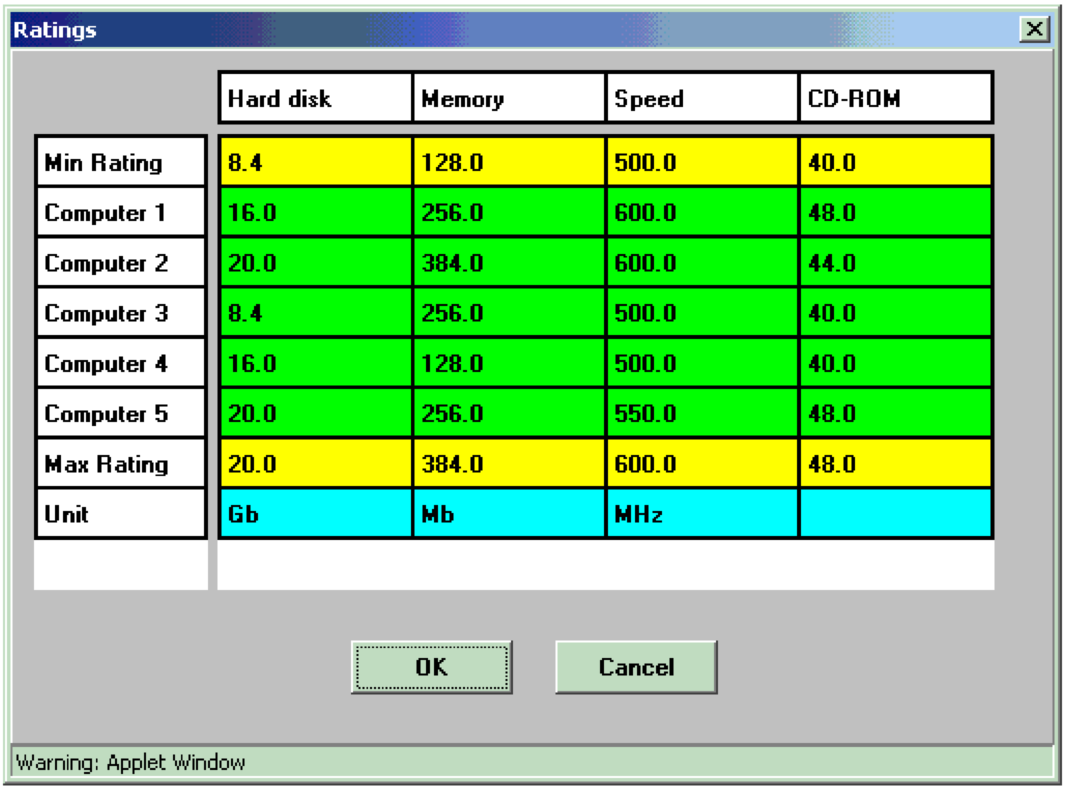


Figure 3.6.1 : A performance matrix constructed with the Web-Hipre software.

See the Job selection case - Specifying attributes [5].

3.7 Desirable properties of the value tree

After the value tree is constructed it is worthwhile to check that it satisfies

[5]http://www.mcda.hut.fi/value_tree/cases/Job/slides/sld023.htm

the following five properties.

1. **Completeness**
   - All relevant objectives should be included in the hierarchy.
   - The set of attributes completely defines the degree to which the overall objective is achieved.

2. **Operationality**
   - Attributes should be meaningful and assessable.

3. **Decomposability**
   - Attributes should be judgementally independent, that is, it should be possible to analyse one attribute at time.

4. **Nonredundancy**
   - The set of attributes should be nonredundant to avoid double counting of the consequences.

5. **Minimum size**
   - The set of attributes should be minimal.

4. Preference elicitation

In the following preference independence of the attributes and an additive value model is assumed. The total value is of the form

$$V(x_1, x_2, \ldots, x_n) = \sum_{i=1}^{n} w_i v_i(x_i)$$

where w_i, $i \in (1,2,\ldots, n)$ corresponds to the relative weight of the attribute X_i.

The weights w_i describe the relative importance of the attributes. That is

Definition 4.1

The weight w_i is associated to the change in total value, when the attribute X_i changes from the worst to the best level.

In most cases, the weights are normalised, in such a way that the sum of the weights equals to the highest value level. Furthermore, it is assumed that single attribute value functions v_i give values ranging from 0 to 1. (Such an assumption can be made since, by definition, positive affine transformations of a value function are strategically equivalent.) For example, in a value scale from 0 to 1

$$\sum_{i=1}^{n} w_i = 1$$

and an alternative with the highest possible performance levels in all attributes will give a total value of 1.

When assessing decision-maker's preferences over the set of attributes, first a single attribute value function has to be constructed for all attributes. Then, the relative weights of the attributes and an aggregated value over the set of attributes can be calculated.

- The Value function elicitation section describes different single attribute value measurement techniques.

- In the Weight elicitation section the problem of weighting and aggregating the values across the attributes is addressed.

4.1 Value function elicitation

The purpose of the value function elicitation is to model and describe the importance and desirability of achieving different performance levels of the given attribute.

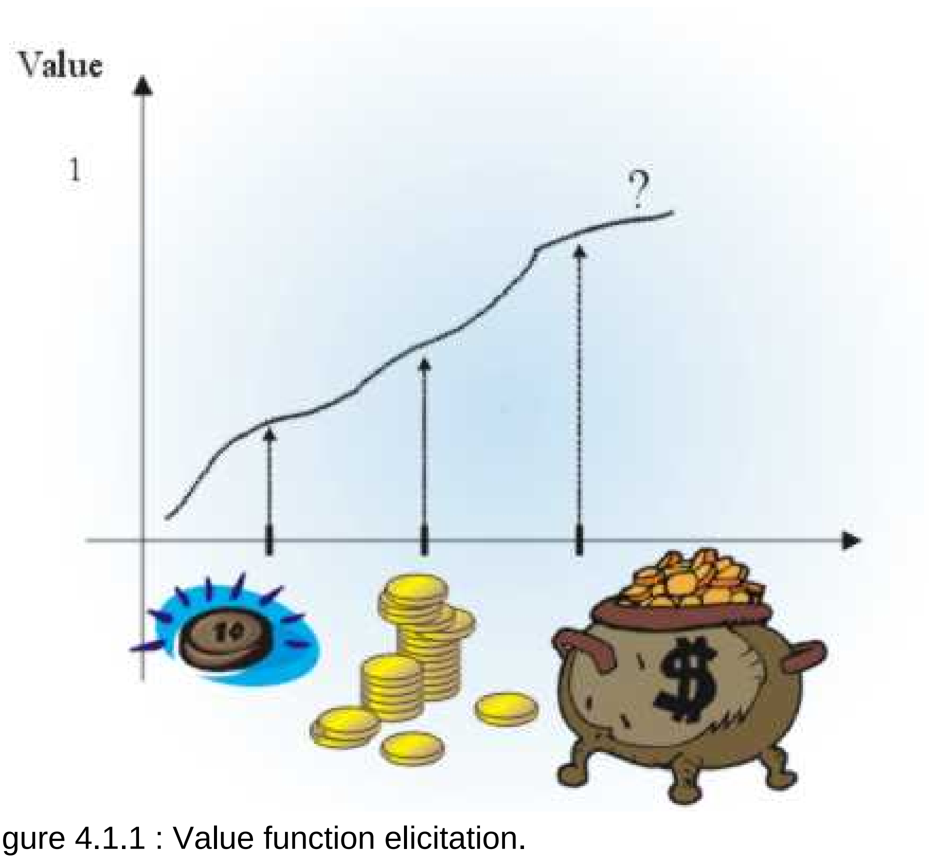


Figure 4.1.1 : Value function elicitation.

In practice, a single attribute value function has to be determined for all attributes. However, in some cases it suffices to determine the values for the attribute levels associated with the alternatives only (i.e. value scores). For example, if changes in the alternatives and corresponding consequences are not expected, eliciting values for other attribute levels would be unnecessary and wouldn't give any further value for the analysis.

When assessing a value function two main phases can be identified: choosing the range, and value elicitation.

4.1.1 Choosing the range

Prior the value elicitation the end points of the range have to be fixed. For example, when assessing preferences over the "working hours per day" attribute, the worst and the best levels have to be determined.

Possible options for the value range of the "working hours per day" attribute:

- **The actual** range is determined by the alternatives with the largest and the smallest number of working hours per day.

- **The acceptable** range is determined by the objects that the decision-maker is willing to consider.

- In **the available** range all available options, not necessarily included in the decision alternatives, should be within the end points.

- **The theoretically feasible** range includes all the alternatives from 0 to 24 working hours per day.

In Figure 4.1.1.1 possible ranges for the working hours attribute are presented.

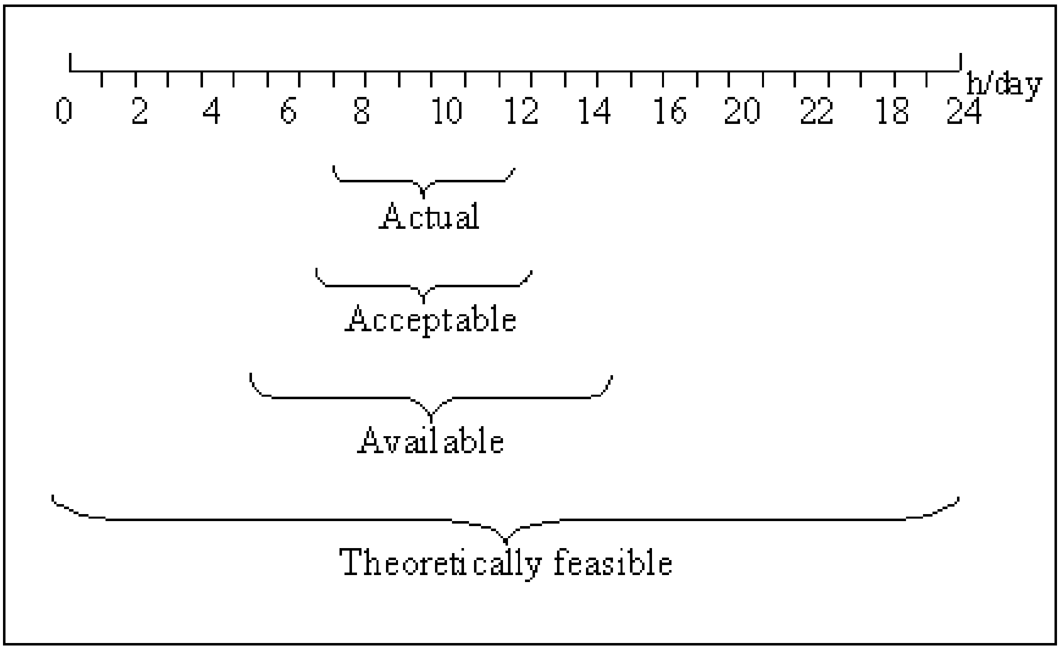


Figure 4.1.1.1 : Possible ranges of the working hours attribute

When selecting the range it should be noted that:

- The choice between the ranges should make no difference to the ranking of the alternatives.
- An advantage of a large range is that it accommodates new decision alternatives more easily if these lie outside the original set. However, the extremes of the range require additional judgements, which may be neither relevant nor helpful for the current decision problem.
- If a large range is chosen the objects are more likely to lie close to another in the middle of the range making the discrimination among them difficult.

4.1.2 Value elicitation

Once the end points are established for each attribute, there are number of different methods that may be used for value elicitation. Table 4.1.2.1 lists the main value measurement techniques and divides them into two main classes: numerical estimation methods and indifference methods.

Numerical estimation	Indifference methods
Direct rating	Difference standard sequence
Category estimation	Bisection
Ratio estimation	
Assessing the form of value function	

Table 4.1.2.1 : Value measurement techniques

- In numerical estimation methods, the DM is presented with an anchored scale and asked to numerically estimate the attractiveness of the given level of an attribute relative to the anchors.
- Indifference methods are based on the assessment of the strength of the value difference. The DM compares pairs of real or hypothetical evaluation objects to each other and revises them until the strength of preference for value differences is equal for both pairs.

Next, the methods listed in Table 4.1.2.1 are explained in detail.

4.1.3 Direct rating

In the direct rating method, the worst and the best consequences with respect to a certain attribute are associated with values of 0 and 100 respectively. Values of the intermediate attribute levels are determined only to the alternatives under consideration.

The steps of the method:

1. Ask DM to rank the alternatives to find the worst and the best one.

2. Clarify the meaning of the scale to the DM by asking reasons for the judgements.

3. Assign a value of 100 to the best and 0 to the worst alternative.

4. Rate the remaining alternatives in between by asking the DM to consider the relative value of the alternatives in such a way that the relative spacing between them reflects the strength of the preferences for one alternative to another.

5. Check the consistency by asking the DM the relative ratings against one other.

6. Continue the iterative value assessment until the DM is comfortable with the values.

In Figure 4.1.3.1 Direct rating is used to measure the value of different levels of the working hours attribute in the Job selection problem. In the value measurement the Web-HIPRE software is used.

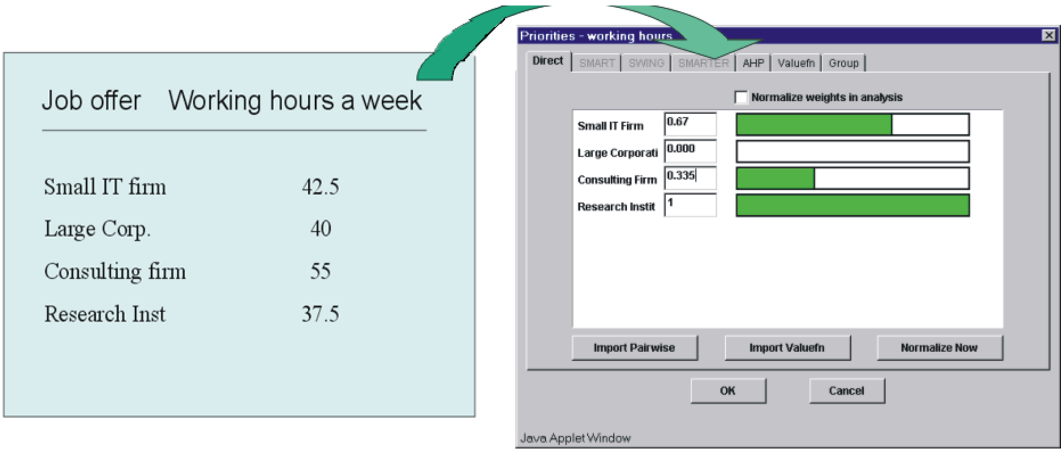


Figure 4.1.3.1 : Direct rating with Web-HIPRE.

Direct rating is most appropriate when

- no commonly agreed scale of measurement exist,
- there is no time or resources to undertake the measurement,
- the performance levels of the attribute can be judged only with subjective measures.

See the Job selection problem - Direct rating[6].

[6]http://www.mcda.hut.fi/value_tree/cases/Job/slides/sld049.htm

4.1.4 Category estimation

Category estimation is a variation of the direct rating technique in which the possible responses of the DM are reduced to a finite number of categories. For each category, a single value from 0 to 1 is given. An example is shown in Table 4.1.4.1.

Category	Poor	Satisfactory	Good
Salary range/€	Less than 2100	2100-2500	More than 2500
Value	0	0.75	1

Table 4.1.4.1 : Salary categories with associated values.

- The advantage of the categorisation is that relatively few preference estimates are needed. Furthermore, the end points are in many cases defined qualitatively, which enables variation in the end points across subjects.
- The downside of the category estimation is that some fine distinctions may get lost. One possibility is to use categorisation as a preliminary screening method for the alternatives and, if necessary, increase the number of categories as the assessment process progresses.

See the Job selection problem - Category estimation[7].

4.1.5 Ratio estimation

In ratio estimation one of the alternatives is presented as a standard and the DM is asked to compare all other alternatives with the standard. Specifically, the DM is asked to state how much more or less valuable an alternative is than the standard, in a ratio sense.

The steps of the method:

1. Choose one of the alternatives as a standard
2. With respect to the selected attribute, compare the other alternatives with the standard by using ratio statements. For example, "42 weekly working hours is 1.5 times less preferable than the standard 37.5 hours per week".
3. Give 1 point to the best alternative
4. Use preference ratios to calculate the values for the other alternatives

The method is based on the assumption that a standard alternative exists and the DM is able to state valuations of the other alternatives in a ratio form. If no standard alternative exists and such an alternative cannot be created, ratios of value differences are compared. In most cases, the ration estimation method is more demanding than the category estimation

[7]http://www.mcda.hut.fi/value_tree/cases/Job/slides/sld052.htm

or direct rating.

Readings

- Torgerson (1958)
- Baird and Noma (1978).

4.1.6 Assessing the form of value function

If an attribute has a numerical scale, one option for the value measurement is the direct assessment of the form of the corresponding single attribute value function. In Figure 4.1.6.1 the assessment of a single attribute value function for the "money" objective is illustrated with the Web-HIPRE software [8].

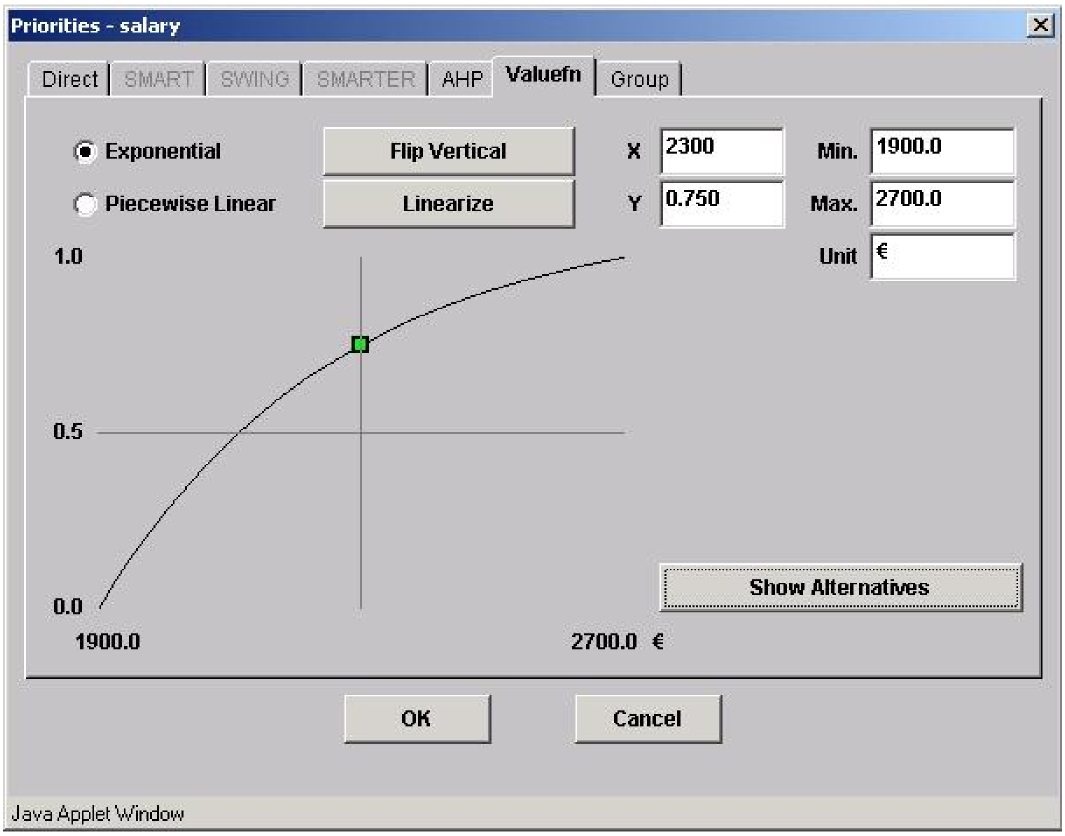


Figure 4.1.6.1 : Assessing a single attribute value function with the Web-Hipre software.

Direct assessment of the form of a value is likely to be difficult and in most cases require expertise or prior knowledge on the subject.

See the Job selection case - Assessing the form of value function[9].

[8]http://www.hipre.hut.fi

4.1.7 Difference standard sequence

In a difference standard sequence method the DM defines attribute levels x_0, x_1, ..., x_n, such that the increments in the strength of preference from x_i to x_{i+1} are equal for all i = 0, ..., n-1. The resulting sequence of attribute levels, equally spaced in value, is called a **standard sequence**.

Since all value steps in the standard sequence are equal we must have

$$v(x_{i+1}) - v(x_i) = k \qquad \forall i = 1, \ldots, n-1$$

where k is a positive constant. Now, k can be chosen freely by taking a corresponding positive affine transformation form v, which is by definition strategically equivalent to the original v. Let k = 1/n and $v(x_0) = 0$. Now

$$v(x_i) = \frac{i}{n} \quad \text{for all i = 0,...,n}$$

In illustration 4.1.7.1 the different standard sequence method is illustrated with an arbitrary value measurement problem.

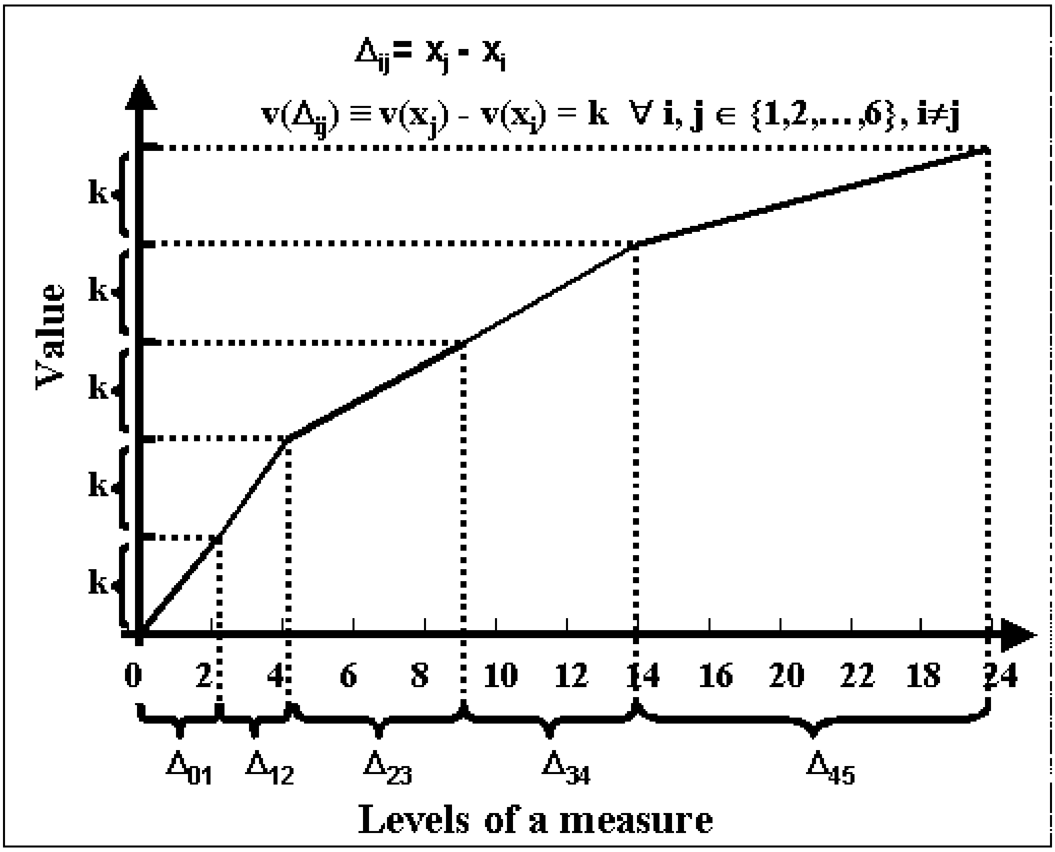


Illustration 4.1.7.1 : Difference standard sequence.

See the Job selection problem - Difference standard sequence[10].

[9]http://www.mcda.hut.fi/value_tree/cases/Job/slides/sld056.htm
[10]http://www.mcda.hut.fi/value_tree/cases/Job/slides/sld042.htm

4.1.8 Bisection

In the Bisection method the DM is presented with two objects and asked to define the attribute level that is halfway between the objects in respect of the relative strengths of the preferences.

1. First, the two extreme points, the least preferred evaluation object x_{min}, and the most preferred evaluation object x_{max} are identified and associated with values

   $$v(x_{min}) = 0$$

   $$v(x_{max}) = 1$$

2. Then, the DM is asked to define a midpoint m_1, for which **(x_{min} m, $_1$)**~ **(m_1,x_{max})**, where (x_i, x_j) indicates the value difference between x_i and x_j, and ~ indicates DM's indifference between the changes in value levels.

   While m_1 is in the middle of the value scale, we must have

   $$v(m_1) = ½\ v(x_{min}) + ½\ v(x_{max}) = 0.5$$

3. For the midpoint m_2 between x_{min} and m_1, and midpoint m_3 between m_1 and x_{max} we have

   $$m_2 = ½\ v(x_{min}) + ½\ v(m_1) = 0.25$$

   $$m_3 = ½\ v(m_1) + ½\ v(x_{max}) = 0.75$$

4. Additional midpoints are determined in a similar way until the value scale is defined with desired accuracy.

In illustation Bisection method 4.1.8.1 the bisection method is illustrated for an arbitrary value measurement problem.

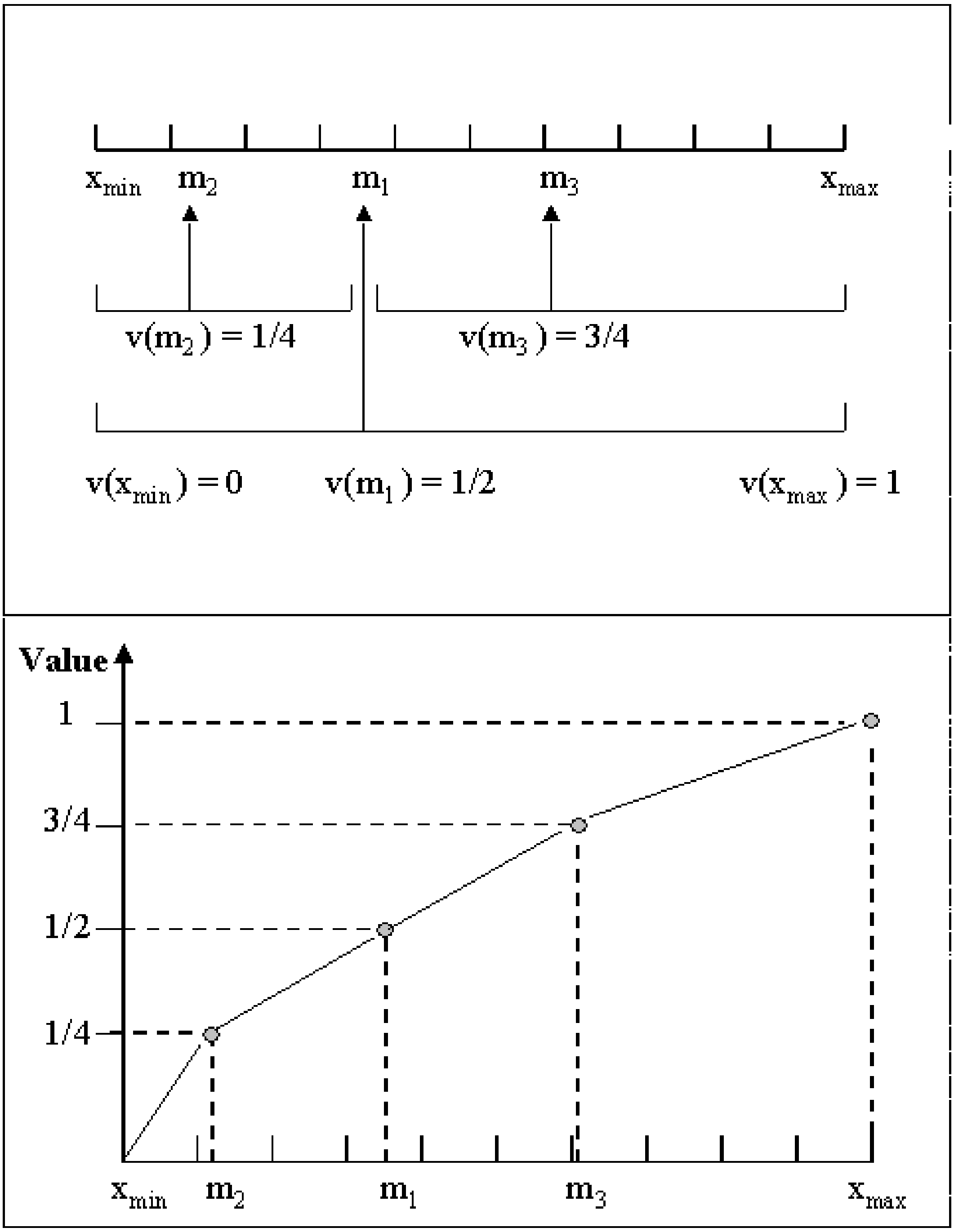


Illustration 4.1.8.1 : Bisection method.

See the Job selection case - Bisection method [11].

Readings

- Pfanzagl (1968)

[11]http://www.mcda.hut.fi/value_tree/cases/Job/slides/sld037.htm

• Torgerson (1958)

4.2 Weight elicitation

There are two ways to determine weights in a value tree:

1. Non-hierarchical weighting:

   Weights are defined for the attributes only.

2. Hierarchical weighting:

   Weights are defined for each hierarchical level separately, and then multiplied down to get the corresponding lower level weights.

In the following illustration 4.2.1, hierarchical and non-hierarchical weighting is illustrated.

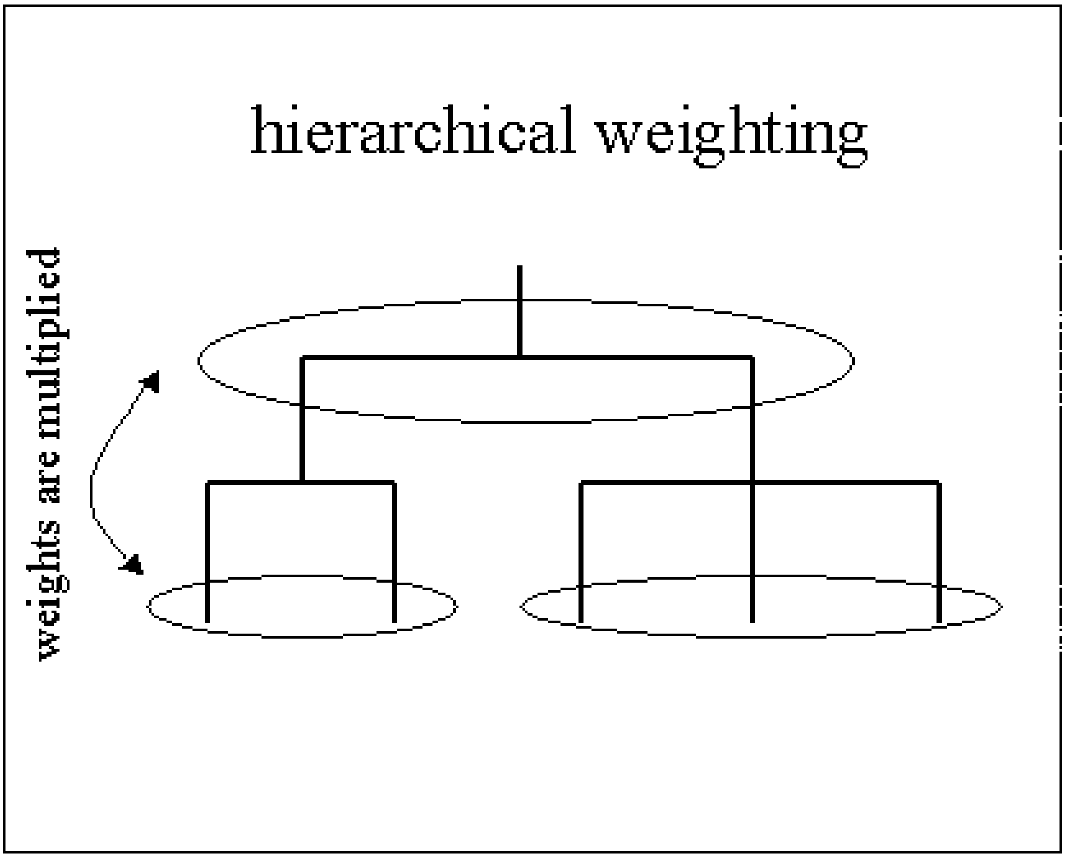


Illustration 4.2.1 : Hierarchical weighting.

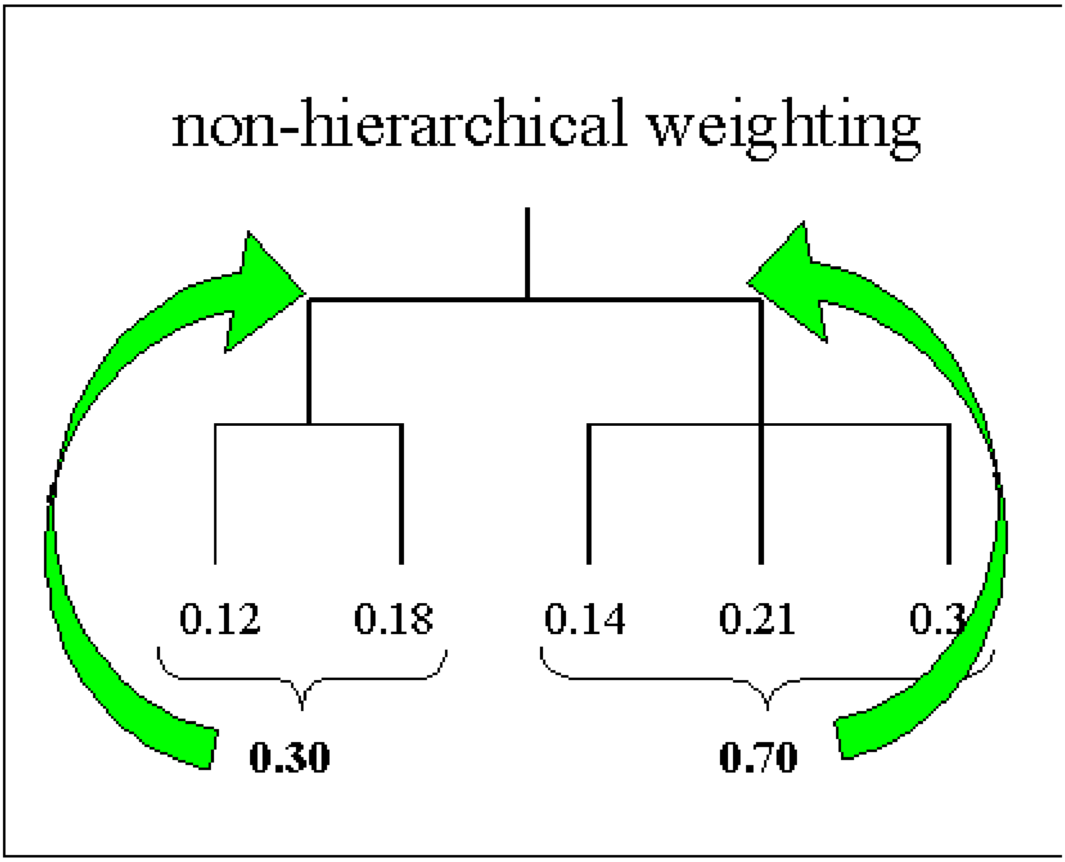


Illustration 4.2.1 : Non-hierarchical weighting.

- In non-hierarchical weighting upper-level weights (objective weights) are not asked, but they can be calculated as a sum of the lower level weights.
- In the additive value model, only the attribute weights are used for determining the overall value of the alternatives.
- Weights of the objectives are used when interpreting the results of the analysis. For example, how much the DM weighted the environmental and economical factors in the aggregate.

In the following sections possible weight elicitation methods are introduced in detail.

4.2.1 SMART - Simple multiattribute rating technique

The steps of the SMART method:

- Firts the DM is asked to give 10 points to the least important at tribute.
- Then the DM is asked to compare the other attributes with the least important one and give them points greater than 10.
- After the comparisons the points are normalised. Thus, the weight of the attribute i is calculated as

$$w_i = \frac{p_i}{\sum_{j=1}^{n} p_j}$$

where p_i corresponds to points given to the attribute (objective) i and n is the total number of the attributes (subobjectives).

See the Job selection case - SMART[12].

4.2.2 Rank based methods

In rank based methods the DM is only asked to define the ranking of the attributes. The weights of the attributes are then calculated by using the mathematical formulae that imply the same order.

Methods are simple and do not require much from the DM; thus they are ideal for a preliminary screening of the alternatives. However, the approach is problematic, while only information on the ranking order of the attributes is used and there are likely to be several weightings implying the same order.

In Table 4.2.2.1 possible ranking based methods for calculating the weight w_j of the attribute j $\in$ (1,..,n) with ranking R_j are presented. Also the formulae for normalised weights w'_j are presented.

Name	w_j	w'_j
Rank sum	$(n+1-R_j)$	$\frac{2(n+1-R_j)}{n(n+1)}$
Rank reciprocal	$1/R_j$	$\frac{1}{R_j \sum_{i=1}^{n} \frac{1}{i}}$
SMARTER (Centroid)	$\sum_{i=R_j}^{n} \frac{1}{i}$	$\frac{1}{n} \sum_{i=R_j}^{n} \frac{1}{i}$
Rank exponent	$(n+1-R_j)^z, z>0$	$\frac{(n+1-R_j)^z}{\sum_{i=1}^{n} (n+1-i)^z}$

[12] http://www.mcda.hut.fi/value_tree/cases/Job/slides/sld061.htm

Table 4.2.2.1 : Ranking based methods.

In Table 4.2.2.2 weights for a tree attribute decision problem are calculated with different ranking based methods.

Method/ Weights	w'_1 n=3	w'_2 n=3	w'_3 n=3
Rank Sum	0.50	0.33	0.17
Rank reciprocal	0.55	0.27	0.18
SMARTER	0.61	0.28	0.11
Rank Exponent, z =0.7	0.45	0.34	0.21

Table 4.2.2.2 : Normalised attribute weights calculated with different ranking based methods.

See the Job selection case - SMARTER[13].

In the ranking based weighting methods splitting the attributes into sub-attributes may change the original weights.

In the following illustration 4.2.2.1 , two upper level attributes are split into sub-attributes. Weights are calculated with the Rank Sum method.

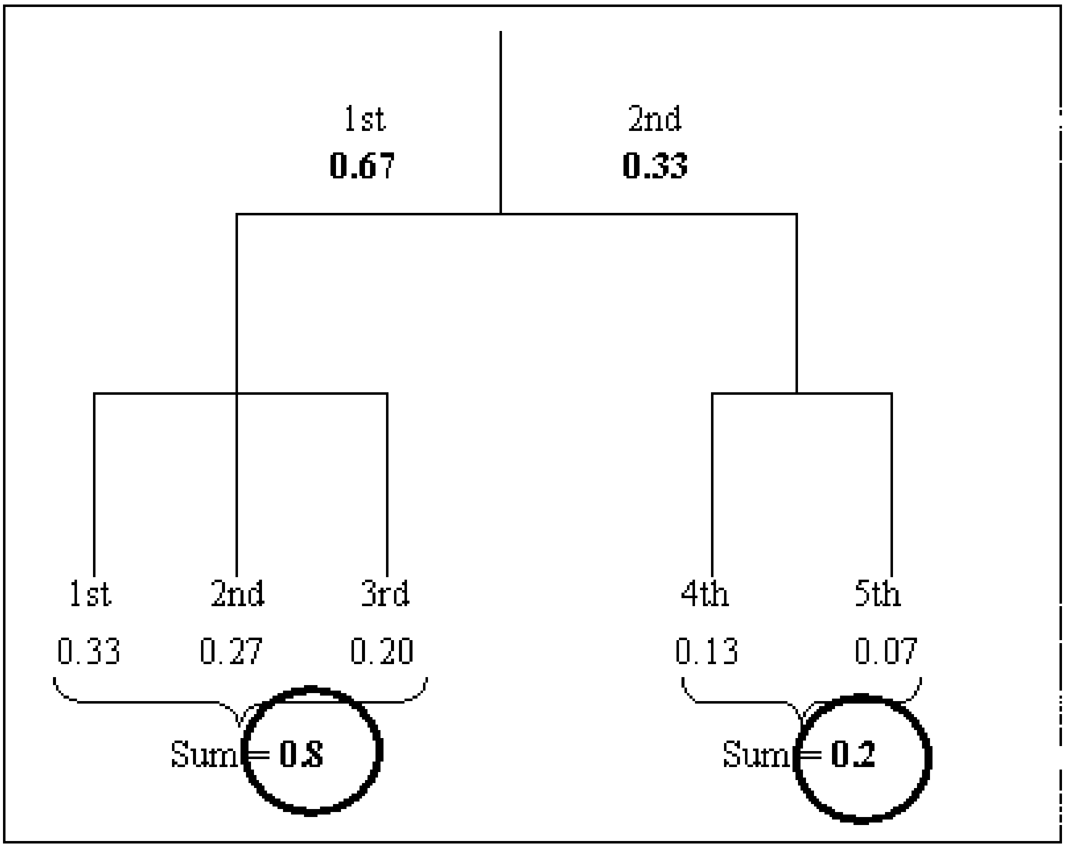


Illustration 4.2.2.1 : Ranking bias

- First, the two upper level attributes are given weights 0,67 and 0,33.

[13]http://www.mcda.hut.fi/value_tree/cases/Job/slides/sld067.htm

- After splitting, the sum of the sub-attributes' weights do not match the original weights.

Readings

- Pöyhönen and Hämäläinen (1998).

4.2.3 SWING

Let x_i^* be the best and x_i^0 the worst outcome of the attribute X_i, $i \in (1,...,n)$. Furthermore let $a^0 = (x_1^0, x_2^0, ..., x_n^0)$ be the worst possible alternative. In the SWING method the DM is asked to consider the alternative a^0 and choose one attribute, say x_i, to be shifted to the highest level x_i^*. The attribute x_i is then given 100 points. Thus we have

$$V(a_0) = 0$$

$$a_i = (x_1^0, x_2^0, \dots, x_{i-1}^0, x_i^*, x_{i+1}^0, \dots, x_n^0)$$

$$V(a_i) = w_i v_i(x_i^*) = w_i = 100$$

Next, the DM is asked to choose another attribute to be shifted to the best level and give it points relative to the first attribute. The procedure is continued until the weights of all attributes are assessed. Finally, the given weights w_i, $i \in (1,..,n)$ are normalised.

See the Job selection problem - SWING[14].

4.3 Imprecise preference statements

- Sometimes the DM doesn't know the exact values for her preferences or the elicitation of the precise values is too complicated and time consuming. In those situations it is possible to use imprecise value statements such as intervals when judging objectives' weights and attributes' performance levels.

- Instead of a single value for an alternative, an interval of the total value is obtained.

- The most preferred solution is determined with dominance assessments or decision rules.

- Imprecise preference statements are also suitable for group decision support, while conflicting views can be captured through aggregate intervals containing group members' individual preference judgements.

[14]http://www.mcda.hut.fi/value_tree/cases/Job/slides/sld064.htm

- In the following a method supporting the imprecise preference statements is introduced.

4.3.1 PRIME - Preference Ratios in Multiattribute Evaluation

- In a PRIME method, positive value differences are used to establish a preference order for the decision alternatives.
- No numerical measurement scale is required.
- Both precise and imprecise ratio statements as well as holistic statements can be used to describe DM's preferences.
- The PRIME Decisions software used in the examples is available for academic use at the web site http://www.hut.fi/Units/SAL/Downloadables/index_fi.html[15].

While the total value function is additive, it is of the form

$$V(x) = \sum_{i=1}^{N} w_i v_i^N(x_i) = \sum_{i=1}^{N} v_i(x_i)$$

where N is the number of attributes in the value tree, x_i is a consequence with regard to the attribute X_i, and $v_i(x_i) = w_i v_i^N(x_i)$ is the weighted normalised value (score) associated with the consequence x_i.

Note that, Equation assumes mutual preferential independence of the attributes.

Let x_i^* and x_i^0 denote the best and the worst consequence measured with the attribute X_i. For a normalized value function we have

$$\begin{cases} v_i^N(x_i^0) = 0 \\ v_i^N(x_i^*) = 1 \end{cases}$$

By assuming that $v_i(x_i^0) = 0$, the total value can be expressed in the form

$$V(x) = \sum_{i=1}^{N} v_i(x_i) = \sum_{i=1}^{N} \left[v_i(x_i^*) - v_i(x_i^0)\right] \left[\frac{v_i(x_i) - v_i(x_i^0)}{v_i(x_i^*) - v_i(x_i^0)}\right] = \sum_{i=1}^{N} w_i v_i^N(x_i)$$

Now it is possible to express attributes' weights and the normalised value function with value differences.

$$w_i = v_i(x_i^*) - v_i(x_i^0)$$

$$v_i^N(x_i) = \frac{v_i(x_i) - v_i(x_i^0)}{v_i(x_i^*) - v_i(x_i^0)} = \frac{v_i(x_i) - v_i(x_i^0)}{w_i}$$

[15] http://www.hut.fi/Units/SAL/Downloadables/index_fi.html

In addition the following normalisation condition must hold

$$\sum_{i=1}^{N} v_i(x_i^*) = \sum_{i=1}^{N} w_i = 1$$

Estimates about the value differences in Equations .2 - .6 are sufficient to support conclusions about the DM's preferences and provide the foundations for preference elicitation in the PRIME method.

In PRIME method preference elicitation is based on

1. Ordinal ranking

   Suppose that the DM prefers consequence x_i^j to x_i^k. Then we must have

   $$v_i(x_i^j) - v_i(x_i^k) > 0$$

   Thus, ordinal ranking implies a set of linear constraints on the single attribute value functions.

2. Ratios of value differences (cardinal ranking)

   Let L and U be the lower and upper limit of the following ratio of value differences

   $$L \le \frac{v(x^j) - v(x^k)}{v(x^l) - v(x^m)} \le U$$

   $$\begin{cases} -v(x^j) + v(x^k) + L(v(x^l) - v(x^m)) \le 0 \\ v(x^j) - v(x^k) - U(v(x^l) - v(x^m)) \le 0 \end{cases}$$

3. Holistic comparisons

   In holistic comparisons ordinal and cardinal ranking techniques are applied to objectives' value functions. For example in holistic ordinal ranking, if the DM prefers the consequence x^1 to x^2 when considering the objective o only, she states that

   $$v_o(x^1) - v_o(x^2) > 0$$

   where v_o is the component value function of the objective o and x^i is the consequence of the alternative i $\in$ {1,2} with regard to the objective o.

PRIME method uses equations 4-6 and linear constrains 7 and 9 to define attributes' (single attribute) value scores and weights with linear programming. For value function elicitation also holistic judgements with Equation 10 may be used.

For example, in direct rating $v_i(x_i^j)$ is positioned relative to the best and the worst consequences x_i^* and x_i^0 to give the correct ratio of value difference.

$$L \le \frac{v_i(x_i^j) - v_i(x_i^0)}{v_i(x_i^*) - v_i(x_i^0)} \le U$$

In Figure 4.3.1.1, direct rating window of the PRIME Decisions software is shown.

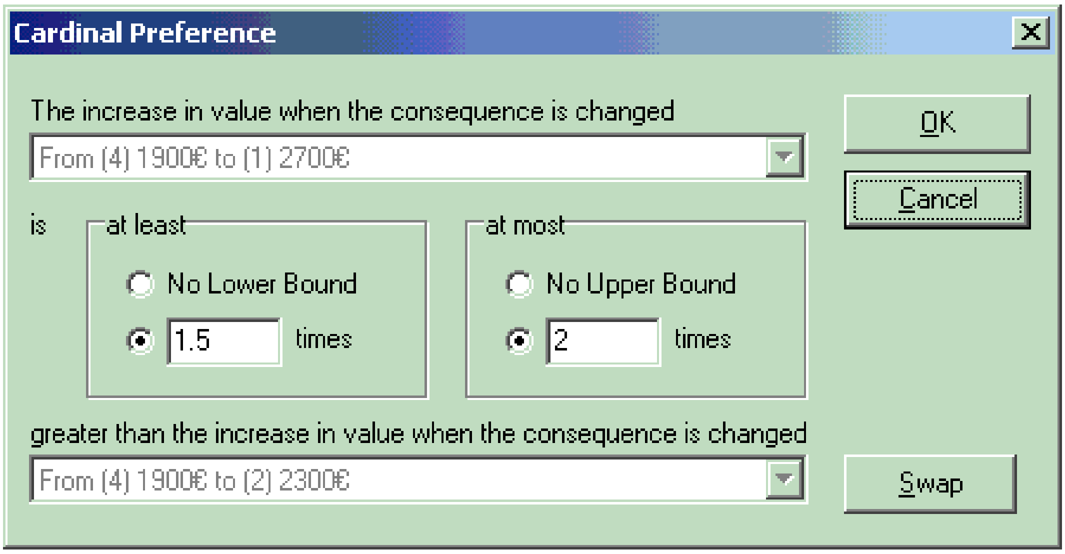


Figure 4.3.1.1 : Direct rating with PRIME Decisions.

Attribute weights can be determined with the following SWING method extension.

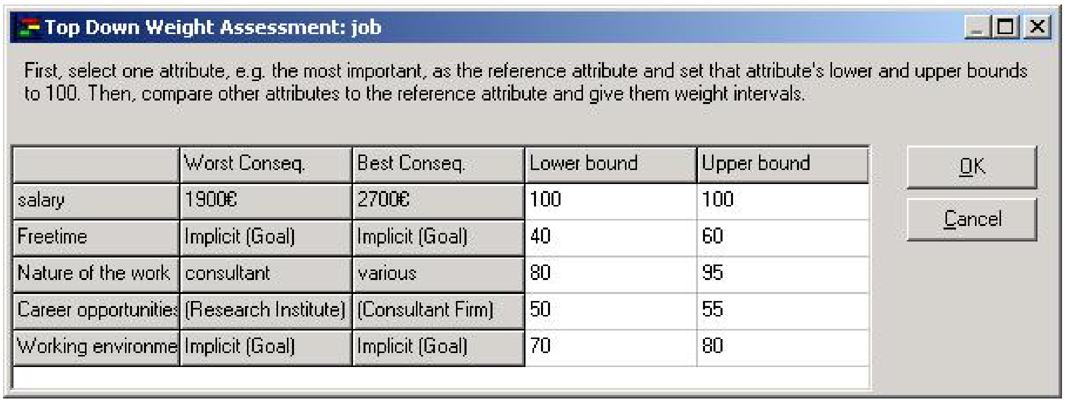
1. Select the most important attribute as a reference and assign 100 points to it.

2. Assign a range of points [L, U] to other attributes in accordance with their perceived importance.

Weight interval judgements leads to the inequalities

$$\frac{L}{100} \le \frac{w_i}{w_{ref}} = \le \frac{U}{100} \Leftrightarrow \frac{L}{100} \le \frac{v_i(x_i^*) - v_i(x_i^0)}{v_{ref}(x_i^*) - v_{ref}(x_{ref}^0)} \le \frac{U}{100}$$

In Figure weight judgements with PRIME Decisions is illustrated.



	Worst Conseq.	Best Conseq.	Lower bound	Upper bound
salary	1900€	2700€	100	100
Freetime	Implicit (Goal)	Implicit (Goal)	40	60
Nature of the work	consultant	various	80	95
Career opportunities	(Research Institute)	(Consultant Firm)	50	55
Working environme	Implicit (Goal)	Implicit (Goal)	70	80

Figure 4.3.1.2 : Weight elicitation with PRIME Decisions.

From the inequalities implied by the DM's judgements the following results are obtained with linear programming.

1. Value intervals for alternatives

$$V(\mathbf{x}) \in \left[\min \sum_{i=1}^{N} v_i(x_i), \max \sum_{i=1}^{N} v_i(x_i) \right]$$

2. Weight intervals for the attributes

$$w_i \in \left[\min\left\{v_i(x_i^*) - v_i(x_i^0)\right\}, \max\left\{v_i(x_i^*) - v_i(x_i^0)\right\}\right]$$

3. Dominance structures

Absolute dominance:

Alternative x^j is preferred to alternative x^k in the sense of absolute dominance if the value intervals of the two alternatives do not overlap. That is, the smallest value of x^j exceeds the largest value of the alternative x^k. Formally,

$$\min \sum_{i=1}^{N} v_i(x_i^j) > \max \sum_{i=1}^{N} v_i(x_i^k)$$

Pairwise dominance:

Alternative x^j is preferred to alternative x^k in the sense of pairwise dominance if

$$\max\left(V(\mathbf{x}^k) - V(\mathbf{x}^j)\right) < 0 \Leftrightarrow$$

$$\max\left(\sum_{i=1}^{N} v_i(\mathbf{x}^k) - \sum_{i=1}^{N} v_i(\mathbf{x}^j)\right) = \max\left(\sum_{i=1}^{N} w_i\left(v_i^N(\mathbf{x}^k) - v_i^N(\mathbf{x}^j)\right)\right) < 0$$

In other words, with any fixed set of weights w_i the (weighted) normalised value of the worst outcome of the alternative x^k is greater than the (weighted) normalised value of the best outcome of x^j.

Note that pairwise dominance is less restrictive criterion than the absolute dominance. Furthermore, pairwise dominance for alternatives x^k and x^j needs to be checked only if

$$\max V(x^j) > \max V(x^k) \geq \min V(x^j) > \min V(x^k)$$

1. If the first inequality in .17 does not hold, regardless of the further refinements in the preference model, the value of the alternative x^j cannot exceed the value of the alternative x^k.

2. If the second inequality doesn't hold x^j dominates the alternative x^k in absolute sense.

3. Finally, if the last inequality doesn't hold there are weights w_i i=1,...,N such that the value of the x^k is greater than the value of x^j.

Consequently, the dominance either follows or is excluded by the value intervals if Equation () is not satisfied.

4. Decision rules

- Maximax:

  the alternative with largest possible value

- Maximin:

  the alternative for which the least possible value is greatest

- Minimax regret:

  the alternative for which the greatest possible loss of value

  $$\max_{j, j \neq k} \left[\max \left(\sum_{i=1}^{N} v_i(x_i^j) - \sum_{i=1}^{N} v_i(x_i^k) \right) \right]$$

  is smallest.

- Central values:

  the alternative for which the midpoint of the value interval # is greatest.

In Figure 4.3.1.3 value intervals for a car selection problem are shown. Figure 4.3.1.4 shows the corresponding Dominance window and Figure Figure 4.3.1.5 the Decision Rules window.

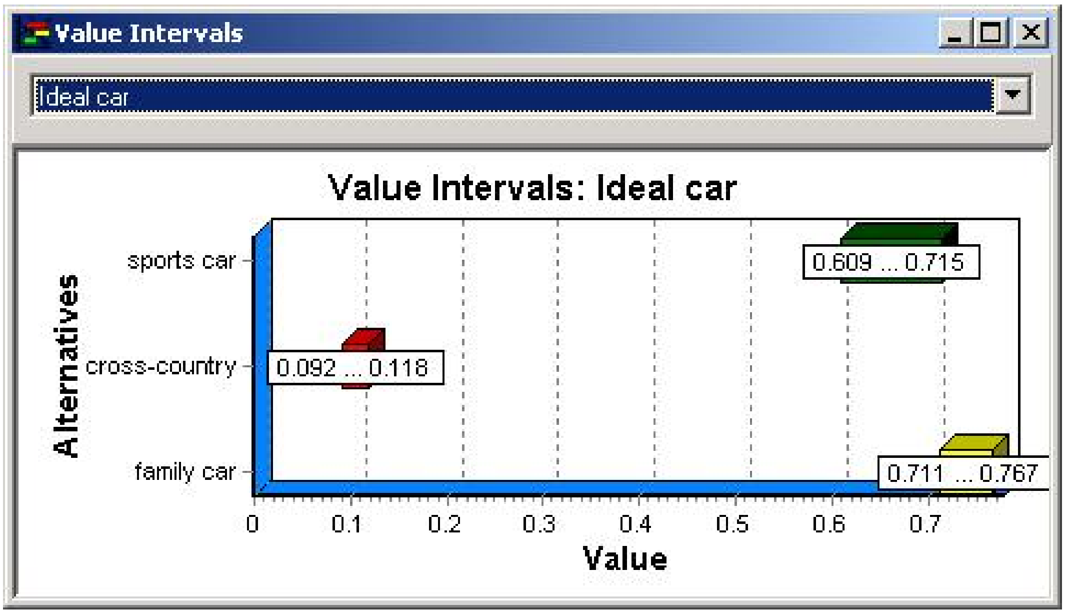


Figure 4.3.1.3 : Value intervals

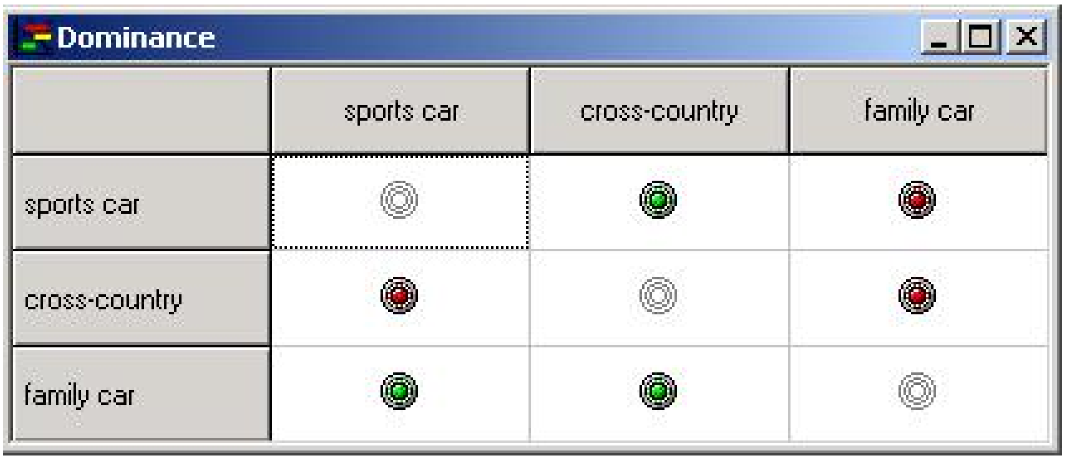


Figure 4.3.1.4 : Dominance structures.

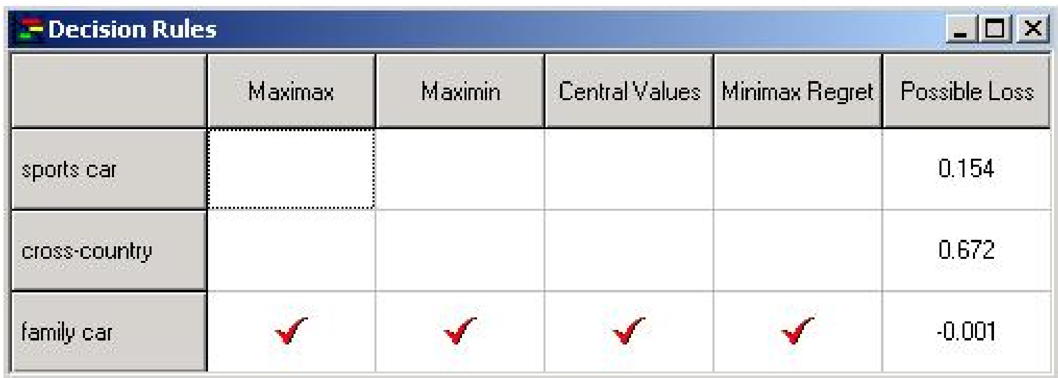


Figure 4.3.1.5 : Decision rules window.

1. Construct the value tree and verify the independence condition.

2. Rank the alternatives with respect to the attributes.

3. Enter a preference statement, either a holistic judgment or a ratio comparison of value differences.

4. Examine the updated absolute and pairwise dominance structures.

5. Iterate through the steps 3 and 4 to reduce the set of nondominated alternatives.

See the Car selection case[16].

Readings

- Gustafsson et al. (2001)
- Salo and Hämäläinen (2001)

4.4 AHP

Analytic Hierarchy Process (AHP) is based on paired comparisons and the use of ratio scales in preference judgements. In the standard form, alternatives are not differentiated from the attributes and objectives but are treated as a bottom level of the hierarchy (as in the Web-HIPRE[17] software). The DM is asked to give the ratio of attributes' (objectives', alternatives') weights

$$r_{ij} = \frac{w_i}{w_j}$$

In comparisons fixed values of r_{ij} associated with verbal statements are used. In Table 4.4.1 a balanced scale is presented.

[16] http://www.mcda.hut.fi/value_tree/cases/Car/slides/
[17] http://www.hipre.hut.fi

Verbal statement	Scale 1-to-9	Balanced
Equally important	1	1.00
-	2	1.22
Slightly more important	3	1.50
-	4	1.86
Strongly more important	5	2.33
-	6	3.00
Very strongly more important	7	4.00
-	8	5.67
Extremely more important	9	9.00

Table 4.4.1 : AHP comparison scale.

Clearly, the selection of comparison scale has an effect on the result. Thus, careful attention should be paid on the scale selection process.

The results of paired comparisons are presented in a **comparison matrix**

$$\mathbf{A} = \begin{bmatrix} r_{11} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots \\ r_{n1} & \cdots & r_{nn} \end{bmatrix}$$

where the elements on the diagonal, r_{ii}, are assumed to be 1. Moreover, only upper triangular matrix is asked and it is stated that

$$r_{ij} = \frac{1}{r_{ji}}$$

The weights are estimated from the estimates w_i by normalising the elements of the eigenvector corresponding the largest eigenvalue of the matrix A. Note that paired comparisons of alternatives with respect to attributes (which corresponds to a single attribute value function elicitation) follow the same procedure.

For n weights (values), the decision-maker gives n(n-1) estimates, thus the estimates might be inconsistent, that is $\exists$ **i, j, k** $\in$ {1,...,**n**} such that

$$a_{ij}a_{jk} \neq a_{jk}$$

It can be shown that A is consistent if and only if $l_{max} = n$, where l_{max} is the largest eigenvalue of the matrix A. An average perturbation value is then given by consistency index

$$CI = \frac{\lambda_{\max} - n}{n-1}$$

and the inconsistency of the weight estimates w_i given by the DM can be measured by consistency ratio index

$$CR = \frac{CI}{CI_{aver}}$$

where CI_{aver} denotes the average of CI over a large number of randomly generated matrices of the order n, with entries derived from the scale 1/K, 1/(K+1),..., 1,..., K-1, K, where K is a positive constant giving the bounds for the real weights.

To be exact, the same comparison scale should be used both in the assessment of the actual comparison matrix and in the generation of the random matrices (Salo and Hämäläinen 1997). An alternative way is to use **scale-invariant consistency measure**

$$CM = \frac{2}{n(n-1)} \sum_{i>j} \frac{\bar{r}(i,j) - \bar{r}(i,j)^{-1}}{\left[1 + \bar{r}(i,j)\right]\left[1 + \bar{r}(i,j)^{-1}\right]}$$

where

$$\bar{r}(i,j) = \max_k a(i,k)a(k,j)$$

is the extended bound of the element a(i, j) in the i:th row and j:th column of the comparison matrix (Salo and Hämäläinen 1997). Generally, consistency ratio or scale invariant consistency measure of 0.2 or less is deemend to be accebtably consistent. If the figure is larger preference statements require further modification.

See the Job selection case - AHP[18] .

Readings

- Saaty (1986)
- Saaty (1994)
- Golden and Wang (1989)

[18]http://www.mcda.hut.fi/value_tree/cases/Job/slides/sld070.htm

- Salo & Hämäläinen (1997)

4.4.1 Rank Reversal

- In the AHP method, a change in the set of alternatives may alter the existing order between the alternatives, even if the original valuations are not changed. The phenomenon is called rank reversal.

- The rank reversal effect is widely seen as a result of the value normalisation, in which the sum of values under an attribute equals one.

- Rank reversal can be avoided by using value functions and normalisation in which the value of 1 is given to the best alternative, 0 to the worst alternative, and others are rated in between.

Readings

- Belton and Gear (1983)

5. Sensitivity analysis

5.1 Purpose

The purpose and the role of the sensitivity analysis in a DA process is describe in the section Value Tree Analysis / Introduction / Sensitivity analysis .

5.2 Dominance

- A decision alternative A is **dominated** by an alternative B if B is at least in one aspect, better than A and in all the other aspects as good as A.

- If a decision alternative is not dominated it is **undominated**.

- Since dominance makes sensitivity analysis unnecessary, it should be analysed prior the sensitivity analysis.

- For example, suppose that you are buying a computer screen and you are only concerned with the price, the size of the screen, and the length of the guarantee. The possible three options are described in Figure 5.2.1 .

Screen	Price	Size	Guarantee
Screen 1	225€	17″	1 years
Screen 2	330€	19″	3 years
Screen 3	320€	19″	1 years

Figure 5.2.1 : Computer screens.

Clearly, screen 1 is dominated by screen 2 and 3, which are both undominated. Thus, there is no need to conduct sensitive analysis for screen 1. If the price of the screen 2 were 300 euros, it would be a dominant alternative and the sensitivity analysis would be unnecessary.

5.3 One-way sensitivity analysis

- In one-way sensitivity analysis objectives' weights, single attribute value functions, or attribute ratings for decision alternatives are varied, one at time, to see how sensitive the model is to those changes.

- The total values of decision alternatives are drawn as a function of the variable under consideration.

- In Figure 5.3.1, sensitive analysis window of the Web-Hipre programme is shown. As the figure shows the overall values of the decision alternatives (Screen 1, 2, 3) are drawn as a function of the weight of the price objective.

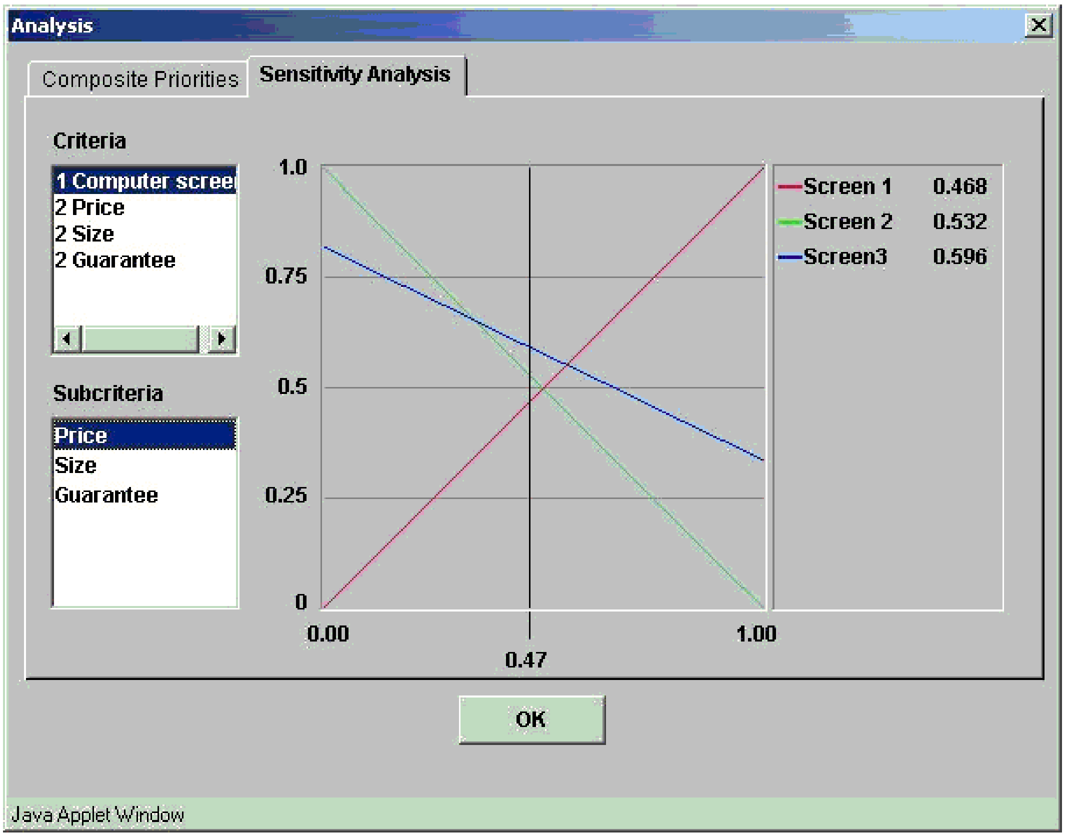


Figure 5.3.1 : One way sensitive analysis with Web-Hipre programme.

- The recommended solution, Screen 3 gives the highest overall value. However, if the weight of the price objective were less than 0.34 Screen 2 would give a higher overall value. Similarly, if the weight of the price objective were higher than 0.55 Screen 1 would become optimal.
- As the current weight of the price is 0.47 at least 17% increase in the weight of the price is required to change the order of the alternatives.
- Whether the model is sensitive to changes in the weight of the price objective or not depends on how precise the current weight estimate is. In other words, how likely the 17% increase is.
- Sensitivity to the changes in the consequences described in the consequence matrix can be analysed in a similar manner.

See the Job selection case - Sensitivity analysis [19].

6. Behavioural issues

DM's preference elicitation may be affected by behavioural issues, biases that cause inconsistencies in weight judgements. In the following some of

[19]http://www.mcda.hut.fi/value_tree/cases/Job/slides/sld081.htm

the biases are introduced.

6.1 Splitting bias

- In non-hierarchical weighting splitting an objective into sub-objectives is likely to increase the weight of the objective.

- Objective weights change because the DMs do not adjust their responses enough to a change in the value tree.

- Figure 6.1.1 shows a value tree and average swing weights from the experiment of Weber, Eisenfähr, and von Winterfeldt (1988). The sum of the lower level weights is shown in parenthesis.

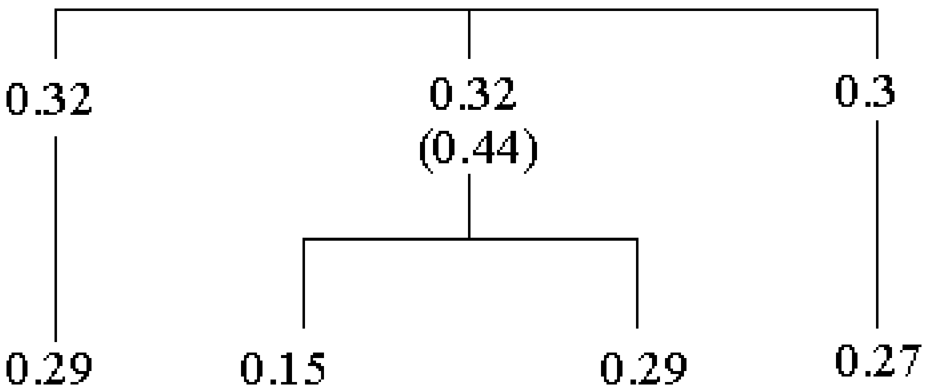


Figure 6.1.1 : Average SWING weights showing the splitting bias (Weber et al. 1988, Figure 3).

- In the experiment subjects evaluated all the attributes at one level simultaneously. As Figure 6.1.1 shows, the average weight of the main objective appeared to be lower than the sum of the weights of the sub-attributes.

- However, average weights do not necessarily describe the behaviour of the individual DM, as shown in Figure 6.1.2.

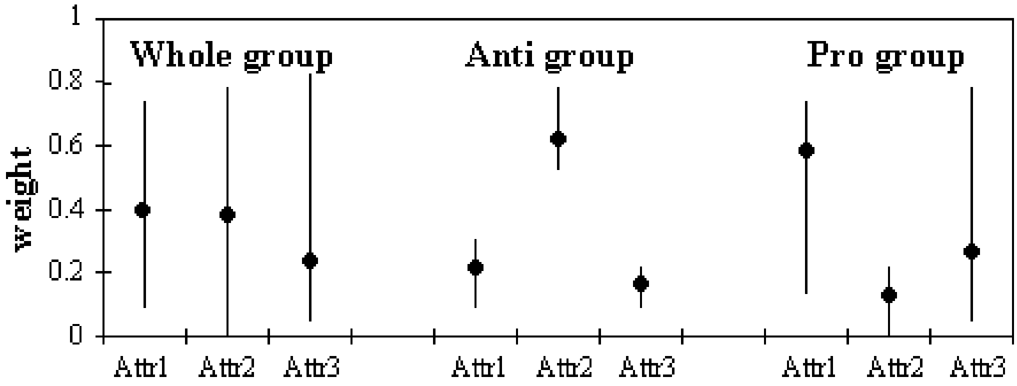


Figure 6.1.2 : The averages and the ranges of weights illustrating the fact that the whole group do not necessarily describe individual opinions (Pöyhönen and Hämäläinen 1998, p. 147, Exhibit 9).

- In recent studies splitting bias has been detected also at the individual level. Furthermore, it has been shown that depending on the structure of the value tree the division of an objective may also **decrease** the weight (Pöyhönen et al 2001).

- Possible explanations:
  - **Availability:** DMs recall more easily those attributes that are presented with more detail. Bringing up the name of an attribute increases the weight of the attribute.

  - People tend to use only certain numbers (even 10s) in their evaluations together with normalisation causes biases in the weights.

- **Note** that in hierarchical weighting splitting bias can not be observed in final weights.

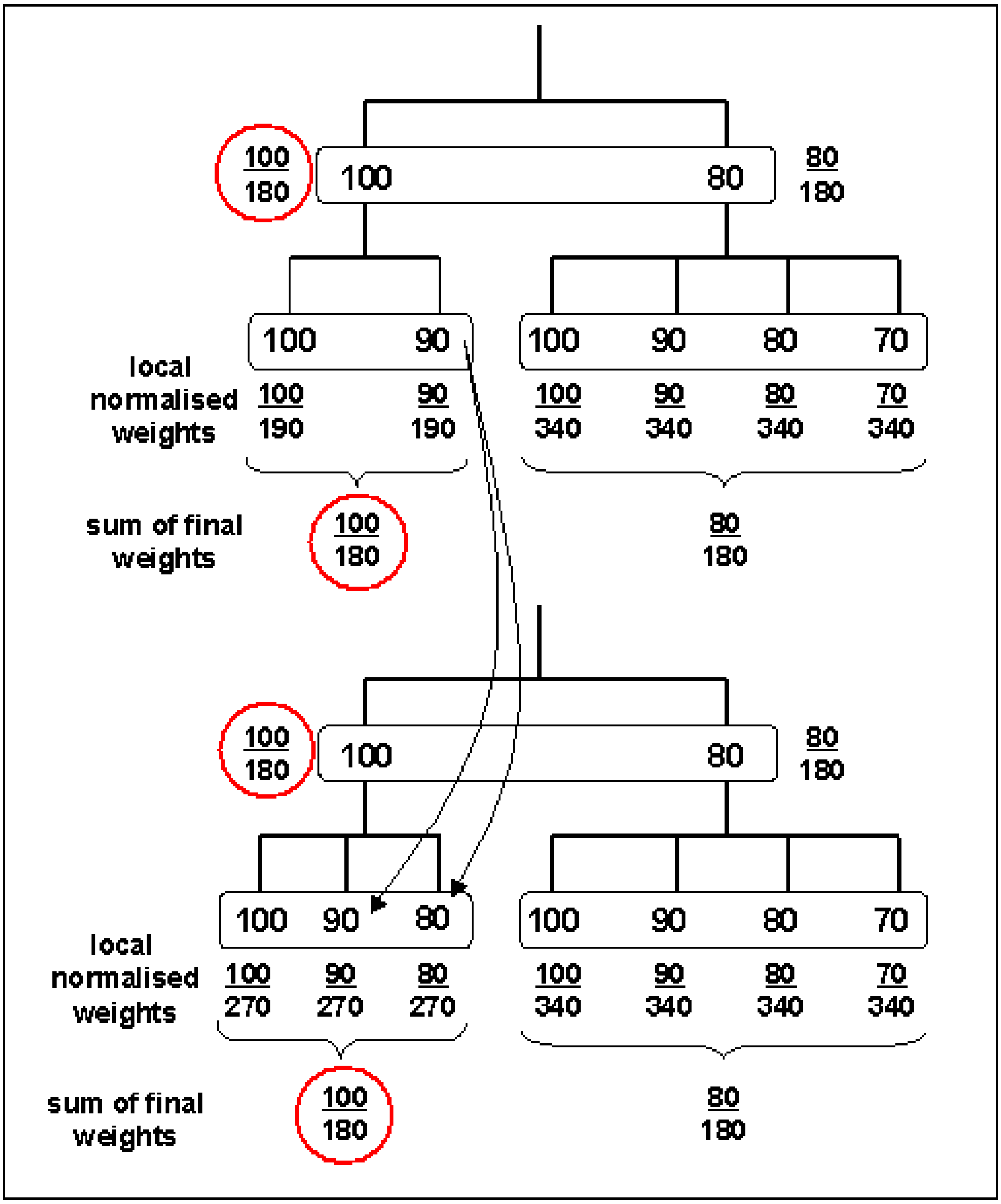


Figure 6.1.3 : Splitting an objective and hierarchical weighting (Pöyhönen et al. 2001, p. 226, Fig. 9).

As Figure 6.1.3 shows, final weights in hierarchical weighting do not show any bias, although the local weights do change in the division.

Readings

- Pöyhönen et al. (2001)
- Weber et al. (1988)

- Borcherding and von Winterfeldt (1988)

6.2 Range effect

Empirical evidence suggests that DMs do not increase the weight of an attribute sufficiently to reflect the corresponding increase in the range of the attribute. For example, ranges of earnings from 1500€ to 3000€ and from 1900€ to 2500€ are given equal weights.

However, when a monotonically increasing conditional value function is normalised on [0, 1], larger range of an attributeshould result in greater weight of the attribute. For a smaller range, smaller weight should be used.

Readings

- Beattie and Baron (1991)
- Fischer (1991)
- Von Nitzsch and Weber (1991)

6.3 The effect of hierarchy

The form of the objectives hierarchy is likely to affect the attribute weights.

1. Different methods are likely to result in different hierarchies. Specifically, there is evidence suggesting that the top-down approach yields steeper value trees with more layers between the top and bottom level (Adelman et al. 1986).

2. The higher in the tree a branch is added, the more weight it is likely to get (Borcherding and von Winterfeldt 1988).

3. Hierarchical weighting leads to higher weight ratios when compared with non-hierarchical weighting (Stillwell et al. 1987). That is, weight ratios in non-hierarchical weighting are closer to 1 than the corresponding ratios in hierarchical weighting.

Readings

- Adelman et al. (1986)
- Borcherding and von Winterfeldt (1988)
- Stillwell et al. (1987)

6.4 Reference point effect

- Depending on the **reference point** the same outcome may be framed as a gain or a loss.
- For example: 5% increase in sales, when
  - 4% was expected
  - 7% was expected
- Weights depend on the status quo of the DM. Losses and disadvantages are likely to have greater impact on preferences than gains or losses. (Tversky and Kahneman 1991).
- Weights derived from equivalent improvements and equivalent losses are not necessarily similar (Shapira 1981).

Readings

- Weber and Borcherding (1992)
- Tversky and Kahneman (1991)
- Shapira (1981)

7. Communicating the results

Presenting the result has an important role in the DA:

- DA is often conducted by the analyst, but the decisions are made by DMs who are responsible for the decisions.
- DMs are not necessarily familiar with the terms and methodologies used in DA.
- In many cases the time is a critical factor for DMs. Thus the results should be easily understood in a reasonably short time period without possibilities to misunderstandings.
- Especially in group decision-making, results should be presented in such a way that there is no room for unnecessary speculations or distorted interpretations.

In many cases graphical presentation of the results is advantageous:

- Graphs allow an easy way to present and understand complex relations and new concepts such as pairwise and absolute dominance etc.

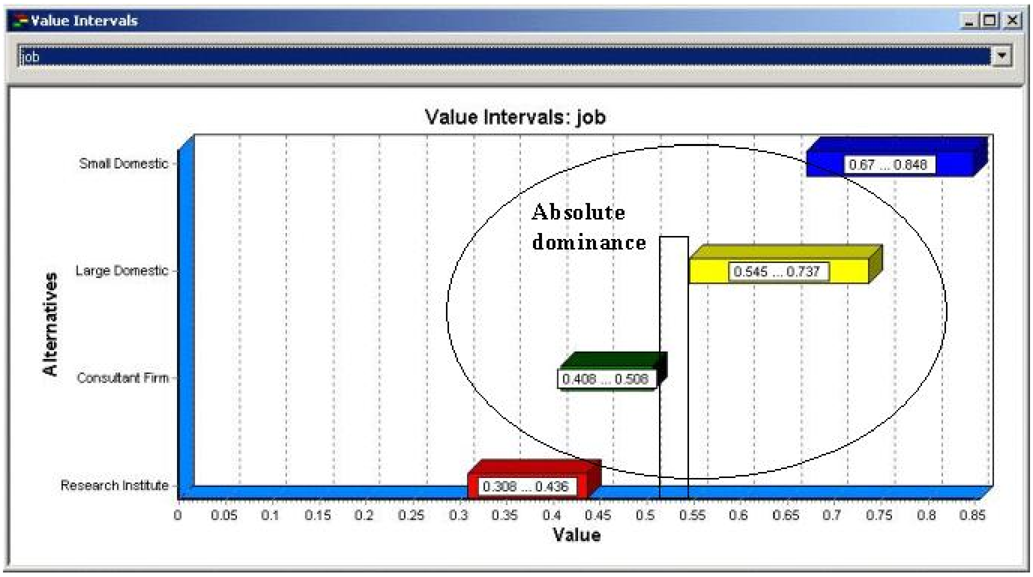


Figure 7.1 : Value intervals in PRIME Decisions and absolute dominance.

- Values of several variables and their proportional magnitudes are more easily detected with bars than numbers. For example, the DM may be interested in what is the relative importance of a single objective when compared with the overall value of an alternative.

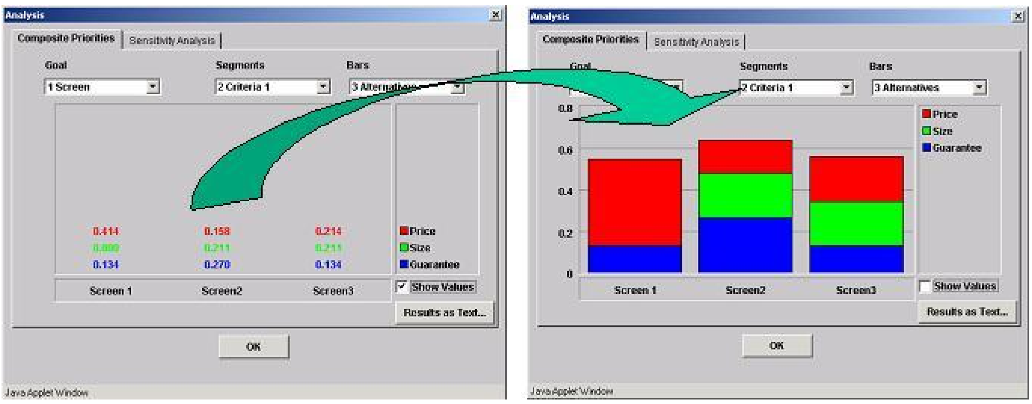


Figure 7.2 : Composite priorities with Web-HIPRE Programme.

- Also the results of sensitivity analysis are more easily understood.

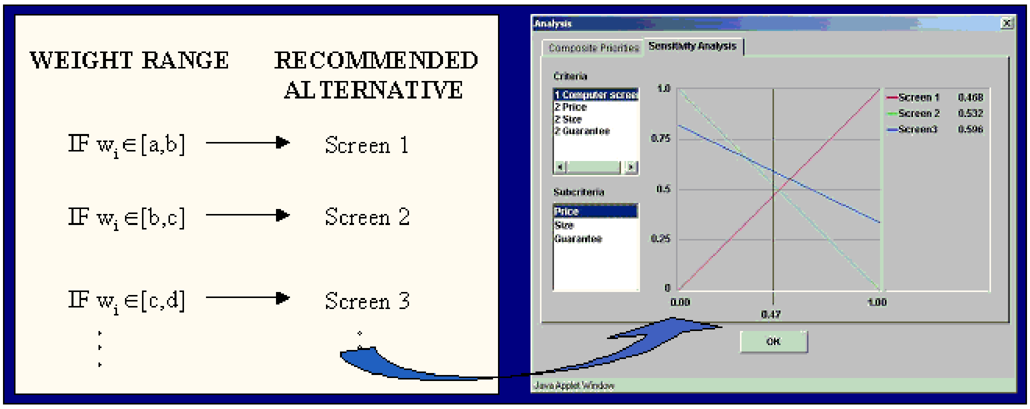


Figure 7.3 : Sensitivity analysis with Web-HIPRE.

- Several software tools with graphical presentation properties are available for DA. A short summary of them is presented in the Software section.

8. Group decision-making

When decision-making process involves several DMs determining singe attribute value functions and weighting the objectives is likely to be difficult. Due the conflicting views, it may be that consensus can not be reached. This section describes how value tree analysis can be applied in group decision-making to aggregate the values of the individual DMs. In the following two possible approaches are presented.

Approach 1

Weighted arithmetic mean method

- Instead of trying to find a commonly agreed value for all parameters in a value tree, the preferences of the individual DMs or groups with similar views are first modelled. (See the preference elicitation section.)

- The overall value of the alternatives is calculated as a weighted sum of the individual values. That is, for the alternative a_j

$$V(a_j) = \sum_{i=1}^{n} w_i v_i(a_j)$$

where n is the number of DMs, $v_i(a_j)$ is the overall value of an alternative a_j valued by DM_i, and k is the total number of the alternatives.

In Figure 8.1 , the approach is summarised.

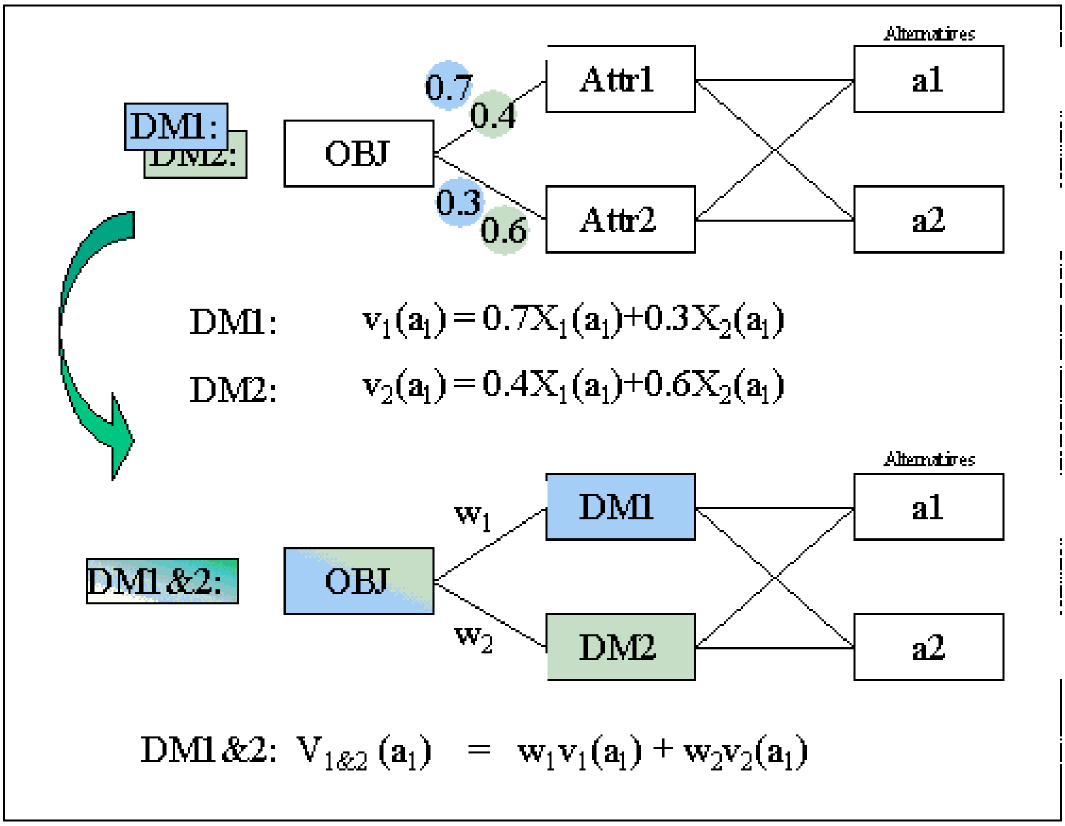


Illustration 8.1 : Group decision-making with value tree analysis.

- As shown in Figure 8.1 , the overall value of the alternatives is calculated with a value tree in which each objective corresponds to the overall value determined by a certain individual value tree. In other words, the objectives in the group model can be associated with the individual value trees.
- Note that also alternatives **a**1, **a**2 are drawn in the model. The same notation is used in the Web-HIPRE software.
- Weights of the groups, or individual DMs, w_i, need not be the same, but they can reflect the level of expertise or power structures, for example.

Approach 2

Imprecise value statements:

- Group preferences are modelled with imprecise preference statements.
- Conflicting views are captured with intervals containing group members' individual preference judgements. In illustration 8.2, an example is given.

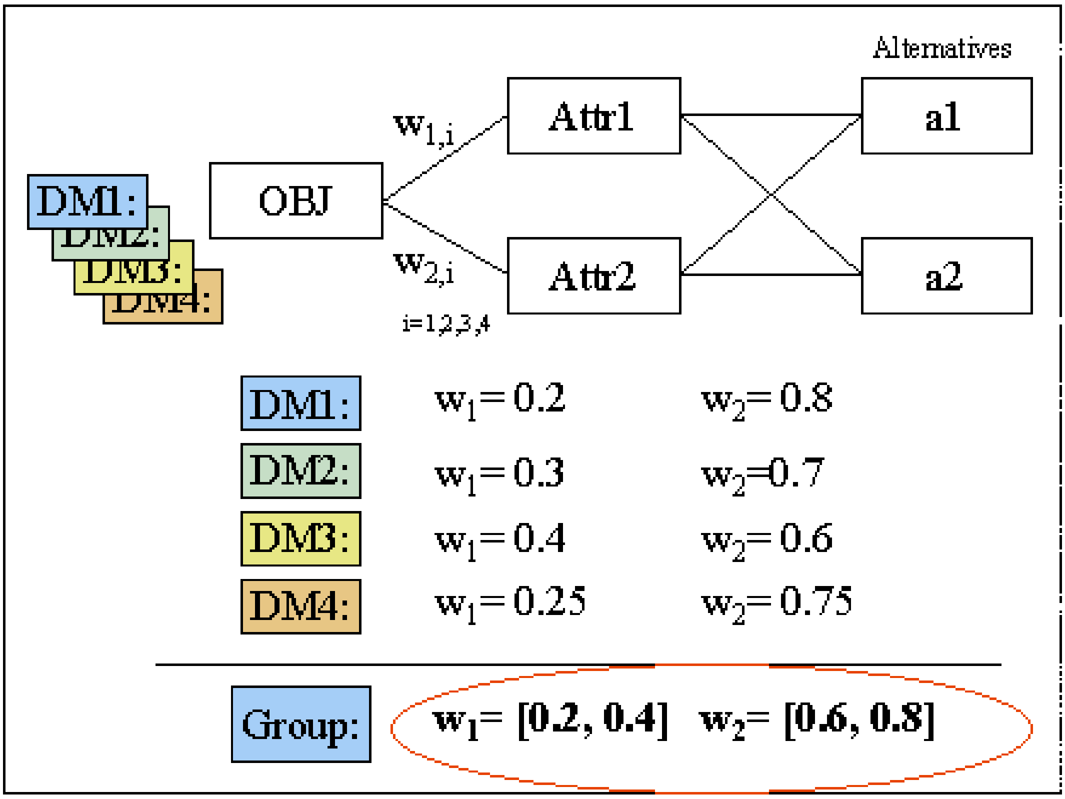


Illustration 8.2 : Determining group weights with imprecise preference statements.

- The use of imprecise preference statements leads to value intervals for the attributes.
- The result is unambiguous only if an alternative dominates the other alternatives in absolute sense. That is, the alternative gives the highest value, and the value interval does not overlap with others.
- If no alternative dominates the others in absolute sense, the recommended solution should be selected among the nondominated solutions.
- It may be that the DM's preference statements need to be refined to reduce the set of the nondominated alternatives.

See the Imprecise preference statements section and the Family selecting a car [20] case.

- Keeney (1976)
- Salo (1995)
- Hämäläinen and Kettunen (1994)

[20] http://www.mcda.hut.fi/value_tree/cases/Car-family/slides/

9. Software

In the following table software tools for DA and value tree analysis are listed.

Software	Vendor	Web site	Suitable for value tree analysis
AIMMS 3	Paragon Decision Technology	www.aimms.com	-
Analytica	Lumina Decision Systems	www.lumina.com	-
Aspen MIMI	Aspen Technology Inc.	www.aspentech.com	-
Criterium Decision Plus 3.0	InfoHarvest Inc.	www.infoharvest.com	-
Crytall Ball 2000 Professional Edition	Decisioneering, Inc.	www.decisioneering.com	-
DATA 3.5	TreeAge Software, Inc.	www.treeage.com	-
Data Interactive	TreeAge Software, Inc.	www.treeage.com	-
Decision Explorer	Baxia Software Ltd	www.banxia.com	-
Decision Hosting	InfoHarvest Inc.	www.infoharvest.com	-
Decision Tools Suite Professional 4.0	Palisade Corporation	www.palisade.com	-
DPL	Applied Decision Analysis LLC	www.adainc.com	-
ELECTRE 3-4	LAMSADE Softwares	www.lamsade.dauphine.fr	-
ELECTRE IS	LAMSADE Softwares	www.lamsade.dauphine.fr	-
ELECTRE TRI	LAMSADE Softwares	www.lamsade.dauphine.fr	-
EQUITY 2 for Windows	Enterprise LSE Ltd.	ww.enterprise-lse.co.uk	-
Expert Choice 2000 Enterprise	EXPERT CHOICE, Inc	www.expertchoice.com	x
EXSYS Developer & Web Runtime	EXSYS, Inc	www.exsys.com	-
Exys Corvid	EXSYS, Inc	www.exsys.com	-
Frontier Analyst	Baxia Software Ltd	www.banxia.com	-
High Priority	Krysalis, Ltd	www.krysalis.co.uk	-
HIPRE 3+	EIA Ltd.	www.eia.fi/hannul/hipre	x
HIVIEW 2 for Windows	Enterprise LSE Ltd.	www.enterprise-lse.co.uk	x
Hugin Professional	Hugin Expert	www.hugin.com	-
Impact Explorer	Baxia Software Ltd	www.banxia.com	-
Joint Gains	Systems Analysis Laboratory	www.decisionarium.hut.fi	-
Logical Decisions for Windows	Logical Decisions	www.logicaldecisions.com	x
Mesa Vista	Mesa Systems Guild, Inc.	www.mesasys.com	-
Netica	Norsys Software Corp.	www.norsys.com	-
On Balance	Krysalis, Ltd	www.krysalis.co.uk	x
On Balance Runtime	Krysalis, Ltd	www.krysalis.co.uk	x
Opinions Online	100GEN Inc.	www.opinions-online.com	-
Policy PC Judgment Analysis Software	Executive Decision Services LLC	www.albany.net/~sschuman/PolicyPC	-
PRIME Decisions	Systems Analysis Laboratory	www.decisionarium.hut.fi	x
Risk Detective	Rhythm Technology, Inc.	www.riskdetective.com	-
TreePlan	Decision Support Services	www.treeplan.com	-
Web HIPRE	Systems Analysis Laboratory	www.decisionarium.hut.fi	x
WINPRE	Systems Analysis Laboratory	www.decisionarium.hut.fi	x

Figure 9.1 : DA software tools and vendors.

Most of the programs and their main properties are listed in Maxwell (2000). The table was last updated 16.1.2002.

Glossary

Additive value model
See the Additive model section.

Alternatives
Ways of achieving objectives. Alternatives may be courses of action, programmes, projects, schemes, systems, or choises in general. Typically the DM has several alternatives and she has to choose one of them.

Attibute
A measure that indicates the level to which an objective is achieved in the given alternatives. (see page 12)

Attribute ratings
See consequences

Consequence
The performance level of an attribute associated to a certain alternative.

Criterion
A standard or means by which a particular choice or course of action can be judged to be more desirable than another one. In literature, the term is also used when referring to attributes or objectives.

Decision alternatives
See alternatives

Decision analyst, DA
See the Parties and roles in decision analysis section.

Decomposition
See the Decomposition section.

DM
Decision making, decision maker.

Dominance
See the Dominance section. See also Absolute dominance and Pairwise dominance.

Goal
A goal is a specific level of an objective to be achieved. In literature the term is also used when referring to the overall objective. (see page 13).

MCDA
Multiple criteria decision analysis (or multiattribute decision analysis, MADA) is an approach and a set of techniques that take explicitly account of multiple, conflicting objectives and attributes, with an objective to help decision makers in identifying a preferred course of action among the set of decision alternatives.

Objective
Objectives are statements of something that one desires to achieve. They are characterised by three features; decision context, object, and direction of preferences. (see page 11)

Objectives hierarchy
See Objectives structures.

Performace level
Possible outcome of an attribute that may appear in one or several alternatives.

Preference independence
See the Preference independence section.

Rating
Specifying the consequences of the alternatives with respect to the given set of attributes.

Value tree
Fundamental objectives hierarchy and attributes (and alternatives) associated with it. (see page 27)

Value function
Assigns a positive number to each consequence indicating the desirability of the consequence . Can be used to derive preferences for the alternatives. (see page 14)

Terms used in the literature

Term used	Terms used in the literature	Example source
attribute	criteria	French (1988)
constructed attribute	subjective scale, subjective index	Huber et al. (1969) Gustafson & Holloway (1975) Keefer & Kirkwood (1978)
decision alternative	option	Winterfeldt & Edwards (1986) Bunn (1984)
measurable value function	value difference function, cardinal value function	French (1988)
natural attribute	measurable attribute	Belton & Stewart (2002)
objective	criteria	Saaty (1994) Belton & Stewart (2002)
ordinal value function	value function, ordinal utility function	Clemen (1995)
overall objective	(top level) goal	Saaty (1994)
performance level	achievement level	Gustafsson et al. (2001) Salo & Hämäläinen (1997)
single attribute value function	partial value function, marginal value function	Belton & Stewart (2002)
single attribute value function elicitation	scoring	Belton & Stewart (2002)
sub-objective	sub-criteria	Saaty (1994)
-	lower level objective	Keeney (1992)
value function	utility function	Clemen (1995)

Figure 10.1 : Terms used in the literature.

References

Adelman, L., Sticha, P. J., Donnell, M. L. (1986). "An Experimental Investigation of the Relative Effectiveness of Two Techniques for Structuring Multiattribute Hierarchies", Organizational Behavior and

Human Decision Processes, 37, 188-196.

Baird, J. C., Noma, E. (1978). Fundamentals of Scaling and Psychophysics. Wiley, New York.

Beattie, J., Baron, J. (1991). "Investigating the Effect of Stimulus Range on Attribute Weight", Journal of Experimental Psychology: Human Perception and Performance 17, 571-585.

Belton, V., Gear, T. (1983). "On a Short-coming of Saaty's Method of Analytic Hierarchies", Omega, 11, 228-230.

Belton, V., T. J. Stewart (2002). Multiple Criteria Decision Analysis An Integrated Approach. Kluwer Academic Publishers, London.

Bernoulli, D. (1738). "Specimen theoriae novae de mensura sortis", Commentari Academica Scientiarum Imperalis Petropolitanae, 175-192; translated in Sommer, L., 1954, Econometrica, 22, 23-36.

Borcherding, K, von Winterfeldt, D. (1988). "The Effect of Varying of Value Trees on Mulltiattribute Evaluations", Acta Psychologica 68, 153-170.

Bunn, D. W. (1984). Applied Decision Analysis, McGraw-Hill, International edition.

Clemen R.T. (1995). Making Hard Decisions: An Introduction to Decision Analysis. Duxbury Press, International edition.

Corner, J. L., C. W. Kirkwood (1991). "Decision Analysis Applications in the Operations Research Literature, 1970-1989" Operations Research, 39, 206-219.

Fischer, G. W. (1995). "Range Sensitivity of Attribute Weights in Multiattribute Value Models", Organizational Behavior and Human Decision Processes, 62(3), 252-266.

Fishburn Peter C. (1970). Utility Theory for Decision Making. John Wiley & Sons, New York, NY.

French Simon 1988. Decision Theory an Introduction to the Mathematics of Rationality. Ellis Horwood Limited, England.

Golden, B. L., Q. Wang (1989). "An Alternate Measure of Consistency" in The Analytic Hierarchy Process- Applications and Studies, eds, Golden , Wasil and Harker, Springer-Verlag, New York.

Gustafson, D. H., D. C. Holloway (1975). "A Decision Theory Approach to Measuring Severity in Illness", Health Services Research, Spring, 97-106.

Gustafsson J., Salo A., Gustafsson T. (2001). "PRIME Decisions: An Interactive Tool for Value Tree Analysis" in Multiple Criteria Decision Making in the New Millenium, eds. Käksalan M., Zionts S., Springer-Verlag, Berlin.

Huber, G. P., V. Sahney, D. Ford (1969). "A Study of Subjective Evaluation Models", Behavioral Science, 14, 483-489.

Hämäläinen , R. P., E. Kettunen (1994). "On-line Group Decision Support by HIPRE 3+ Group Link" Proc. of the 3rd International Symposium on the Analytic Hierarchy Process, Washington, D.C., July 11-13, 547-557.

Keefer, D. L., C. W. Kirkwood (1978). "A Multiobjective Decision Analysis: Budget Planning for Product Engineering", Journal of Operational Research Society, 29, 435-442.

Keeney Ralph R. (1992). Value-Focused Thinking: A Path to Creative Decisionmaking. Harvard University Press, Cambridge, Massachusetts, London, England.

Keeney Ralph L. (1976). "A Group Preference Axiomatization with Cardinal Utility", Management Science, 23(2), 140-145.

Keeney Ralph L., Howard Raiffa, (1976). Decisions with Multiple Objectives: Preferences and Value Tradeoffs. Wiley, New York.

Maxwell D. T. (2000). "Decision Analysis: Aiding Insight V", ORMS Today, 27(5), 28-35.

Pfanzagl, J. (1968). Theory of Measurement. Wiley, New York.

Pöyhönen, M., Vrolijk, H., Hämäläinen, R. P. (2001). "Behavioral and Procedural Consequences of Structural Variation in Value Trees", European Journal of Operational Research 134, 216-227.

Pöyhönen, Hämäläinen (1998). "Notes on the weighting biases in value trees", Journal of Behavioral Decision Making 11,139-150.

Saaty Thomas L. (1994). "Fundamentals of Decision Making and Priority Theory with the Analytic Hierarchy Process". RWS Publications, Pittsburgh, USA.

Saaty Thomas L. (1986). "Axiomatic Foundation of the Analytic Hierarchy Process", Management Science, 32(7).

Salo A. (1995). "Interactive Decision Aiding for Group Decision Support", European Journal of Operational Research, 84, 134-149.

Salo A., Hämäläinen R. P. (2001). "Preference Ratios in Multiattribute Evaluation (PRIME) - Elicitation and Decision Procedures under Incomplete Information", IEEE Transactions on Sysytems, Man, and Cybernetics. Part A: Systems and Humans, 31(6).

Salo, A., Hämäläinen R.P. (1997). "On the Measurement of Preferences in the Analytic Hierarchy Process", Journal of Multi-Criteria Decision Analysis, 6, 309-319.

Shapira, Z. (1981). "Making Trade-offs Between Job Attributes", Organizational Behavior and Human Decision Making, 28, 331-335.

Stillwell, W. G., von Winterfeldt, D., John, R. S. (1987). "Comparing Hierarchical and Nonhierarchical Weighting Methods for Eliciting Multiattribute Value Models", Management Science, 33, 442-450.

Torgerson, W. S. (1958). Theory and Methods of Scaling. Wiley, New York.

Tversky, A., Kahneman, D. (1991)." Loss Aversion in Riskless Choice: A Reference-Dependent Model", The Quarterly Journal of Economics, 106(4), 1039-1061.

Ulvila, J. W. , R. V. Brown (1982). "Decision Analysis Comes of Age", Harvard Business Review, September-October 1982, 130-141.

Weber, M., Borcherding, K. (1992). "Behavioral Influences on Weight Judgments in Multiattribute Decision Making" European Journal of Operational Research, 67, 1-12.

Weber, M., Eisenführ, F., von Winterfeldt, D. (1988). "The Effects of Splitting Attributes on Weights in Multiattribute Utility Measurement", Management Science 34, 431-445.

von Nitzsch, R., Weber, M. (1993). "The Effect of Attribute Ranges on Weights in Multiattribute Utility Measurements", Management Science 39, 937-943.

von Winterfeldt, D., W. Edwards (1986). Decision Analysis and Behavioral Research. Cambridge University Press, International edition.